1. Consider an unpaired two-sample comparison utilizing independent, continuous measurements. The Wilcoxon test involves \( T = \sum_{i \in \{1st \ sample\}} R_i \), where \( R_i \) is the rank of the \( i \)th observation taken relative to the combined sample. What is \( E(T) \) on the null hypothesis that the samples are governed by the same distribution?

2. Two random variables, \( X \) and \( Y \), are conditionally independent given the parameter \( \theta \), on which a Bayesian analyst places a prior distribution \( p(\theta) \).

   (a) What distribution records the analyst’s uncertainty about \( \theta \) after \( X = x \) is observed?

   (b) Prior to observing \( Y \), the analyst has a ‘prior’ for \( \theta \) which the distribution in part (a).

   What is the analyst’s posterior distribution after observing \( Y = y \)?

   (c) Show that the result in (b) is the same, regardless of which order we make observations.

3. An observation \( Y \) is modeled as having a Gamma distribution with known shape \( a > 0 \) and unknown rate \( \theta > 0 \). That is;

\[
f(y; \theta, a) = \frac{\theta^a y^{a-1} \exp(-\theta y)}{\Gamma(a)}
\]

for \( y > 0 \). If uncertainty about \( \theta \) is recorded in another Gamma distribution, with shape \( \alpha \) and rate \( \beta \), determine:

   (a) The posterior distribution of \( \theta \) given \( Y = y \).

   (b) The prior predictive distribution for \( Y \).

4. Reconsider problem (1) in the trivial case of one observation in each sample, denoted \( X \) and \( Y \), say. So

\[
T = \begin{cases} 
1 & \text{if } X < Y \\
2 & \text{if } X > Y 
\end{cases}
\]

Suppose that \( X \) and \( Y \) are both normally distributed with common variance (say one) but different means (i.e., the null hypothesis is not true). Find an expression for \( E(T) \) in terms of the cumulative distribution function \( \Phi() \) of a standard normal variate.