Discussion 4 for STAT 371, Spring 2008

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Review

1. Probability tree

2. Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) > 0$$

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

$$P(A) = P(A \cap B) + P(A \cap B^c) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

3. Law of total Probability

$$P(A) = \sum_{i=1}^{n} P(A \cap B_i) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

where B_i are a partition of S(sample space)

4. Bayes Theorem (simple)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B \cap A^c) + P(B \cap A)}$$

5. Bayes Theorem (extension)

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)}$$

Examples

1. Let $P(A) = 0.3$, $P(B) = 0.2$, and $P(A \cap B) = 0.1$. Find $P(A|B)$, $P(B|A)$, $P(A^c|B)$, and P($B^c|A$).

2. It is known that 10% of football players use steroid. A manufacturer claims that its drug test will detect steroid use (that is, show positive for an athlete who uses steroids) 95% of the time. The test is also positive for 5% of non-steroids users. Your friend on the football team has just tested positive. The probability that he uses steroids is:

3. Suppose there are two bowls full of cookies. Bowl 1 has 10 chocolate chip cookies and 30 plain cookies, while bowl 2 has 20 of each. Fred picks a bowl at random, and then picks a cookie at random. We may assume there is no reason to believe Fred treats one bowl differently from another, likewise for the cookies. The cookie turns out to be a plain one. How probable is it that Fred picked it out of bowl 1? Intuitively, this should be greater than half since bowl 1 contains the same number of cookies as bowl 2, yet it has more plain.

   We can clarify the situation by rephrasing the question to "Whats the probability that Fred picked bowl 1, given that he has a plain cookie?" The event A is that Fred picked bowl 1, and the event B is that Fred picked a plain cookie. To compute $P(A|B)$, we first need to know:

   - $P(A)$, or the probability that Fred picked bowl 1 regardless of any other information. Since Fred is treating both bowls equally, it is (   ).
   - $P(B)$, or the probability of getting a plain cookie regardless of any other information. Since there are 80 total cookies, and 50 of them are plain, the probability of selecting a plain cookie is (   ).
   - $P(B|A)$, or the probability of getting a plain cookie given Fred picked bowl 1. Since there are 40 cookies in bowl 1 and 30 of them are plain, the probability is (   ).
     `http://en.wikipedia.org/wiki/Bayes_theorem`