

# Chapter 10

## One Multicategory Response

### 10.1 Study Suggestions

In retrospect, I do not like the advice I give in the text for the visual presentation of an unordered categorical response. Insisting, as I do, that the categories be ordered by frequency of responses can lead to confusion when, for example, the most common response is “None of the above,” or “About the same.” If no confusion will result, as, for example, in the winning lottery numbers example, I advocate following my advice. For situations in which my advice leads to confusion, the categories may be presented in a natural sequence, but you should remember not to place any significance on the shape of the bar chart.

The chi-squared goodness of fit test is very famous. It was developed by Karl Pearson to provide a mathematical evaluation of the important work by Gregor Johann Mendel. Despite its fame, modern statisticians are largely indifferent to the test. The test’s main problems are twofold. First, one population inference is inherently less interesting than comparing two populations. Second, the applicability of the test is severely restricted by the requirement of having  $k$  special values of interest. The goodness of fit test does, however, provide a good motivation and framework for the more useful test of Chapter 11.

I use the example of the Wisconsin lottery daily game on pages 345 and 346 to illustrate one of my main concerns with the teaching of introductory statistics: If the only tool students have for analyzing one multicategory response is the chi-squared test, then should we be surprised if they apply this test indiscriminately? This example in the text illustrates the power and flexibility of computer simulation to answer questions that the standard theory

cannot handle. (Note: I wrote letters to the heads of the lottery commissions in every state that has a lotto-type game, asking if they perform any statistical analyses of the winning numbers to check for fairness. Approximately 25 heads wrote back and said that they did, and all but one of these 25 states uses the chi-squared goodness of fit test, without any adjustments.)

The computer simulation experiment for the Wisconsin lottery daily game also provides you with the first example of how to study robustness. This topic is discussed further in Chapter 15.

### 10.2 Solutions to Odd-Numbered Exercises

#### Solutions for Section 10.2

1. For the round and yellow peas,  $\hat{p} = 315/556 = 0.567$ . The 95 percent confidence interval is

$$0.567 \pm 1.96 \sqrt{\frac{0.567(0.433)}{556}} =$$
$$0.567 \pm 0.041 = [0.526, 0.608].$$

- For the round and green peas,  $\hat{p} = 108/556 = 0.194$ . The 95 percent confidence interval is

$$0.194 \pm 1.96 \sqrt{\frac{0.194(0.806)}{556}} =$$
$$0.194 \pm 0.033 = [0.161, 0.227].$$

- For the wrinkled and yellow peas,  $\hat{p} = 101/556 = 0.182$ . The 95 percent confidence

interval is

$$0.182 \pm 1.96 \sqrt{\frac{0.182(0.818)}{556}} =$$

$$0.182 \pm 0.032 = [0.150, 0.214].$$

For the wrinkled and green peas,  $\hat{p} = 32/556 = 0.058$ . The 95 percent confidence interval is

$$0.058 \pm 1.96 \sqrt{\frac{0.058(0.942)}{556}} =$$

$$0.058 \pm 0.019 = [0.039, 0.077].$$

11. First, compute  $\sqrt{7.00} = 2.65$ . The area to the right of 7.00 under the chi-squared curve with one degree of freedom equals twice the area to the right of 2.65 under the standard normal curve. Using Table A.5, the former representation gives an area smaller than 0.01, but from Table A.2, the latter representation gives an area equal to  $2(0.0040) = 0.0080$ . These two answers are consistent (0.0080 is smaller than 0.01), but the more precise answer from Table A.2 is preferred.

### Solutions for Section 10.3

1. The new analysis gives  $\chi^2 = 4.43$  with two degrees of freedom. The approximate P-value is the area to the right of 4.43 under the chi-squared curve with two degrees of freedom. From Table A.5, this area is between 0.10 and 0.50. Thus, the data are not statistically significant.

### 10.3 Exam Questions

1. Elisa performs a chi-squared goodness of fit test on a categorical variable with six categories. She obtains  $\chi^2 = 5.00$  for the value of her test statistic. Compute the value of the (approximate) P-value.
2. Help Jose perform his chi-squared goodness of fit test by finding the values of  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$  in the following table. (Note: His sample size is  $n = 100$ .)

$p_0$	$O$	$E$
$a$	16	12
0.17	15	17
0.14	15	14
0.26	21	$b$
0.21	$c$	21
$d$	16	$e$

3. Criticize the following (the computations are correct). A researcher throws a die 600 times and obtains 120 sixes. The researcher then decides to test the null hypothesis that the probability of a six equals one-sixth versus the alternative that it is larger than one-sixth. The researcher obtains  $z = 2.19$  and  $P = 0.0143$  and concludes that the die is not balanced.
4. Help Sonja perform her chi-squared goodness of fit test by finding the values of  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$  in the following table. (Note: Her sample size is  $n = 100$ .)

$p_0$	$O$	$E$
$a$	35	20
0.35	22	$b$
0.15	$c$	15
$d$	18	$e$

5. In one of his studies on pea plants, Mendel hypothesized that four types of plants—round yellow, round green, wrinkled yellow, and wrinkled green—are equally likely to occur. To test this hypothesis, he grew 110 plants and obtained 31, 26, 27, and 26 plants, respectively, of these four types. Analyze these data.
6. Members of the family of chi-squared curves are characterized by a number called the
- Sample size
  - Mean
  - Standard deviation
  - Degrees of freedom
  - Number of cells
7. Use Table A.5 of the text to find the area to the right of 10.64 under the chi-squared curve with six degrees of freedom.

8. Use Table A.5 of the text to find the area to the right of 15.09 under the chi-squared curve with four degrees of freedom.
9. Use Table A.5 of the text to find the area to the right of 16.92 under the chi-squared curve with 10 degrees of freedom.
10. Use Table A.2 of the text to find the area to the right of 4.41 under the chi-squared curve with one degree of freedom.
11. A response has four possible values, labeled 1, 2, 3, and 4. A theory states that the probabilities of occurrence of categories 1–4 are in the ratio 5:3:2:1. If the theory is correct, what are the values of  $p_1$ ,  $p_2$ ,  $p_3$ , and  $p_4$ ?
12. A response has five possible values, labeled 1, 2, 3, 4, and 5. A theory states that
- Response 1 is twice as likely as response 3.
  - Responses 2 and 4 are equally likely.
  - Response 3 is twice as likely as response 5.
  - Response 2 is three times as likely as response 5.

If the theory is correct, what are the values of  $p_1$ ,  $p_2$ ,  $p_3$ ,  $p_4$ , and  $p_5$ ? (Hint: If the theory is correct,  $p_5$  is the smallest of the probabilities.)

13. Teri wants to investigate whether a die is balanced. She casts the die 12 times and obtains the following frequency distribution.

Outcome	Frequency
1	0
2	3
3	5
4	0
5	1
6	3

Compute the value of the chi-squared goodness of fit statistic for these data.

14. Kramer reasons as follows. If each human baby has a 50 percent chance of being a female, then the number of females in a two-child family should have a binomial distribution with  $n = 2$  and probability of success equal to 0.5.

Using the formula of Chapter 5, Kramer next determines that for such families the probability of 0 female is 0.25, the probability of 1 female is 0.50, and the probability of 2 females is 0.25. (These determinations by Kramer are correct, and you need not verify them.)

To test his reasoning, Kramer selects a random sample of 100 families from the population of families having exactly two children. Kramer obtains 70 families with 1 female, 15 families with 0 female, and 15 families with 2 females. Use these data to test Kramer's hypothesized Bin(2,0.5) distribution.

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To test his reasoning, Kramer selects a random sample of 100 families from the population of families having exactly two children. Kramer obtains 70 families with 1 female, 15 families with 0 female, and 15 families with 2 females. If these data are used to test Kramer's hypothesized Bin(2,0.5) distribution, one finds that the chi-squared goodness of fit test yields an approximate P-value that is smaller than 0.01. Kramer concludes, "Either the two genders are not equally likely or the genders of the successive births are not independent." Provide a different—plausible—explanation of the pattern in Kramer's data.

## 10.4 Solutions to Exam Questions

1.  $0.10 < \mathbf{P} < 0.50$ .
2.  $a = 0.12$ ,  $b = 26$ ,  $c = 17$ ,  $d = 0.10$ , and  $e = 10$ .
3. There are two problems with the analysis. First, the researcher chooses the alternative after looking at the data (“The researcher *then* decides ...”). Second, a goodness of fit test is better than (possibly) focusing on an extreme result.
4.  $a = 0.20$ ,  $b = 35$ ,  $c = 25$ ,  $d = 0.30$ , and  $e = 30$ .

5. The goodness of fit test yields

$$\chi^2 = 0.618.$$

From Table A.5,  $0.50 < \mathbf{P} < 0.90$ .

6. (d).
7. Area = 0.10.
8. Area < 0.01.
9.  $0.05 < \text{Area} < 0.10$ .
10. Area = 0.0179.
11.  $p_1 = 5/11$ ,  $p_2 = 3/11$ ,  $p_3 = 2/11$ , and  $p_4 = 1/11$ .
12.  $p_1 = 4/13$ ,  $p_2 = 3/13$ ,  $p_3 = 2/13$ ,  $p_4 = 3/13$ , and  $p_5 = 1/13$ .
13.  $\chi^2 = 10$ .
14.  $\chi^2 = 16$ , and the approximate P-value is smaller than 0.01. Kramer’s hypothesis should be rejected.
15. Kramer’s data contain a shortage of families with 0 or 2 females. If many families desire at least one child of each gender, then families with one boy and one girl will tend to stop having children at two, but families with all boys or all girls will continue to have babies, and not belong to his population.