Solutions to the Practice First Midterm Exam

January 16, 2015

1. (a) The completed table is below. The frequencies are obtained by counting; the relative frequencies are the frequencies divided by \( n = 50 \); the densities are the relative frequencies divided by \( w \), which is, 1, 0.5, 0.5 and 2, for the four class intervals, respectively.

<table>
<thead>
<tr>
<th>Class Int.</th>
<th>Freq.</th>
<th>Rel. Freq.</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00–1.00</td>
<td>14</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>1.00–1.50</td>
<td>17</td>
<td>0.34</td>
<td>0.68</td>
</tr>
<tr>
<td>1.50–2.00</td>
<td>12</td>
<td>0.24</td>
<td>0.48</td>
</tr>
<tr>
<td>2.00–4.00</td>
<td>7</td>
<td>0.14</td>
<td>0.07</td>
</tr>
</tbody>
</table>

(b) \( g = 1.42 - 0.875 = 0.545 \) and \( h = 1.42 + 0.875 = 2.295 \).

(c) Eight values are smaller than \( g \) and 5 values are larger than \( h \); hence, 37 values fall between them. The answer is \( 37/50 = 0.74 \); i.e., 74%.

(d) The smallest deviation is \( 0.11 - 1.42 = -1.31 \) and the largest deviation is \( 3.63 - 1.42 = 2.21 \).

2. (a) All three could be true. Option (i) would occur, for example, if the numbers in the two center positions are 49.8 and 50.0. Option (ii) would occur, for example, if the numbers in the two center positions are 49.7 and 50.3. Finally, option (ii) would occur, for example, if the numbers in the two center positions are 49.9 and 50.2.

(b) The area of the rectangle is \( 7.5(0.003067) = 0.023 \); this is the relative frequency of the interval. The frequency is

\[ 1000(0.023) = 23. \]

(c) The answer is 5.0. According to the Empirical Rule, the interval \( 50.0 \pm s \) will contain approximately 68% of the data. The options given for \( s \) translate into this interval being: 2, 4, 6 or 8 rectangles. By inspection, the center 4 rectangles look possible; the others have either not enough data or too much data.

(d) i. M. This would be true, for example, if the smallest observation is 32.6 and the largest is 67.4. This would be false, however, if the smallest observation is 32.6 and the largest is 67.3.

ii. T. This must be true because the two center positions for the new data set are the same as the two center positions for the original data set.

3. (a) Trials 1 and 6 are assigned to treatment 1 and all other trials are assigned to treatment 2. Following our notation, the assignment is 1,6.

(b) The computations are:

\[ \bar{x} = (5 + 20)/2 = 12.5; \]

\[ \bar{y} = (10 + 15 + 0 + 15 + 10)/5 = 10; \]

and \( u = 12.5 - 10 = 2.5 \).

(c) Assignment 1,4 puts the two smallest responses on treatment 1; hence, it gives the smallest possible value of \( u \):

\[ \bar{x} = (5 + 0)/2 = 2.5; \]
\[
\bar{y} = \frac{(10 + 15 + 15 + 20 + 10)}{5} = 14; \\
\text{and } u = 2.5 - 14 = -11.5.
\]

(d) As argued above, the assignment is 1,4.

(e) Both assignments 3,6 and 5,6 put the two largest responses on treatment 1, yielding:
\[
\bar{x} = \frac{(15 + 20)}{2} = 17.5; \\
\bar{y} = \frac{(5 + 10 + 15 + 0 + 10)}{5} = 8; \\
\text{and } u = 17.5 - 8 = 9.5.
\]

(f) As argued above, the assignments are 3,6 and 5,6.

4. (a) I recommend that you create the table for the clone-enhanced study. Begin with:

<table>
<thead>
<tr>
<th>Trial</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tr. 1:</td>
<td>5</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>20</td>
<td>?</td>
</tr>
<tr>
<td>Tr. 2:</td>
<td>?</td>
<td>10</td>
<td>15</td>
<td>0</td>
<td>15</td>
<td>?</td>
<td>10</td>
</tr>
</tbody>
</table>

Next, apply the constant treatment effect of +10 to each trial:

<table>
<thead>
<tr>
<th>Trial</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tr. 1:</td>
<td>5</td>
<td>20</td>
<td>25</td>
<td>10</td>
<td>25</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Tr. 2:</td>
<td>-5</td>
<td>10</td>
<td>15</td>
<td>0</td>
<td>15</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

We can now proceed as in problem 2:
\[
\bar{x} = \frac{(25 + 20)}{2} = 22.5; \\
\bar{y} = \frac{(-5 + 10 + 15 + 0 + 10)}{5} = 6; \\
\text{and } u = 22.5 - 6 = 16.5.
\]

(b) Assignment 3,5 puts the two largest responses on treatment 1, yielding:
\[
\bar{x} = \frac{(25 + 25)}{2} = 25; \\
\bar{y} = \frac{(-5 + 10 + 0 + 10 + 10)}{5} = 5; \\
\text{and } u = 25 - 5 = 20.
\]

(c) As argued above, the assignment is 3,5.

5. (a) The ranks are (1 + 11)/2 = 6 for ‘Low;’ 
(12+24)/2 = 18 for ‘Medium;’ and (25+34)/2 = 29.5 for ‘High.’ Thus,
\[
r_1 = 2(6) + 4(18) + 4(29.5) = 202; \text{ and } \\
r_2 = 9(6) + 9(18) + 6(29.5) = 393.
\]

(b) The mean ranks are:
\[
\bar{r}_1 = 202/10 = 20.2; \text{ and } \\
\bar{r}_2 = 393/24 = 16.4.
\]

Thus, treatment 1 gives larger responses.

6. (a) \[\mu = n_1(n + 1)/2 = 10(35)/2 = 175.\]

(b) First, we note that \[t_1 = 11, t_2 = 13 \text{ and } t_3 = 10.\]

Thus, \[\sum(t_i^3 - t_i)\] equals 
\[(11^3 - 11) + (13^3 - 13) + (10^3 - 10) = 4494.
\]

Thus the variance is:
\[
\sigma^2 = \frac{n_1 n_2 (n + 1)}{12} - \frac{n_1 n_2 (4494)}{12(n(n - 1))} = \\
\frac{10(24)(35)}{12} - \frac{10(24)(4494)}{12(34)(33)} = \\
700 - 80.11 = 619.89.
\]

(c) The standard deviation is \[\sigma = \sqrt{619.89} = 24.90.\]

7. We get:
\[
\bar{x} = (18 + 12 + 24)/3 = 18; \\
\bar{y} = (9 + 12 + 21)/3 = 14; \text{ and } \\
u = 18 - 14 = 4.
\]

Sorting the combined data we get:
Thus,\[r_1 = 2.5 + 4 + 6 = 12.5\]and\[r_2 = 1 + 2.5 + 5 = 8.5.\]

8. **Answer**: The first step in problems 8–10 is to transfer the actual data to the clone-enhanced table, as I do below.

\[
\begin{array}{ccccccc}
\text{Sbjt:} & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Tr. 1:} & 18 & 12 & & & & \\
\text{Tr. 2:} & 9 & 12 & & & &
\end{array}
\]

Next, we use the assumption that the Skeptic is correct to fill in the six blank spaces in the table above.

\[
\begin{array}{ccccccc}
\text{Sbjt:} & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Tr. 1:} & 18 & 9 & 12 & 12 & & \\
\text{Tr. 2:} & 18 & 9 & 12 & 12 & 21 & 24
\end{array}
\]

9. As in Problem 10, we start with the actual data:

\[
\begin{array}{ccccccc}
\text{Sbjt:} & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Tr. 1:} & 18 & & 12 & & & \\
\text{Tr. 2:} & 9 & 12 & & & &
\end{array}
\]

Next, we use the assumption of a constant treatment effect to complete the table.

\[
\begin{array}{ccccccc}
\text{Sbjt:} & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Tr. 1:} & 18 & 13 & 16 & 12 & 25 & 24 \\
\text{Tr. 2:} & 14 & 9 & 12 & 8 & 21 & 20
\end{array}
\]

10. As in Problem 10, we start with the actual data:

\[
\begin{array}{ccccccc}
\text{Sbjt:} & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Tr. 1:} & 18 & 12 & & & & \\
\text{Tr. 2:} & 9 & 12 & & & &
\end{array}
\]

Next, we use the assumptions above to complete the table.

\[
\begin{array}{ccccccc}
\text{Sbjt:} & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Tr. 1:} & 18 & 11 & 14 & 12 & 16 & 24 \\
\text{Tr. 2:} & 16 & 9 & 12 & 17 & 21 & 29
\end{array}
\]

11. (a) The frequency of \((U \geq 0.55)\) is

\[54 + 23 + 6 + 1 = 84.\]

Thus, the relative frequency is 0.084.

(b) The frequency of \((U \leq 0.25)\) is 1000 minus the frequency of \((U > 0.25)\); the latter of which is:

\[71 + 54 + 23 + 6 + 1 = 155.\]

Thus, the frequency we want is

\[1000 - 155 = 845,\]

and the relative frequency is 0.845.

(c) We need the frequency of \((U \leq -0.65)\) added to the frequency of \((U \geq 0.65)\). The first is:

\[2 + 6 + 17 = 25,\]

and the second is:

\[23 + 6 + 1 = 30.\]

Thus, the total frequency is \(25 + 30 = 55\) and the relative frequency is 0.055.
(d) For 0.084, we get
\[ 0.084 \pm 3 \sqrt{\frac{0.084(0.916)}{1000}} = 0.084 \pm 0.026. \]
The other answers are: 0.845 ± 0.034 and 0.055 ± 0.022.

12. (a) The ranks are:
\( (1 + 11)/2 = 6 \) for Low;
\( (12 + 20)/2 = 16 \) for Med; and
\( (21 + 29)/2 = 25 \) for High.

Thus,
\[ r_1 = 2(6) + 3(16) + 4(25) = 160 \text{ and } \]
\[ r_2 = 9(6) + 6(16) + 5(25) = 275. \]

(b) We must compare the mean ranks:
\[ \bar{r}_1 = 160/9 = 17.78 \text{ and } \]
\[ \bar{r}_2 = 275/20 = 13.75. \]

Thus, treatment 1 gives larger responses.

(c) First, note that the \( t_i \)'s are 11, 9 and 9.

Thus,
\[ \sum (t_i^3 - t_i) = (11^3 - 11) + 2(9^3 - 9) = 2760. \]

The first term in the variance is
\[ \frac{9(20)(30)}{12} = 450. \]

The second term in the variance is
\[ \frac{9(20)(2760)}{12(29)(28)} = 50.985. \]

Thus, the variance is
\[ 450 - 50.985 = 399.015, \]

and the standard deviation is
\[ \sqrt{399.015} = 19.975. \]

13. (a) The P-value is \( P(U \geq u) = 1 - P(U < u) = 1 - 0.1346 = 0.8654. \)

(b) The P-value is \( P(U \leq u) = P(U < u) + P(U = u) = 0.1346 + 0.0670 = 0.2016. \)

(c) By symmetry the P-value equals twice the smaller of the previous two answers:
\[ 2(0.2016) = 0.4032. \]

14. (a) The P-value is \( P(U \geq u) = P(U > u) + P(U = u) = 0.1643 + 0.0527 = 0.2170. \)

(b) The P-value is \( P(U \leq u) = 1 - P(U > u) = 1 - 0.1643 = 0.8357. \)

(c) By symmetry the P-value equals twice the smaller of the previous two answers:
\[ 2(0.2170) = 0.4340. \]