There are two versions of each problem. All problems on a given page are the same version. For example, because problems 1 and 2 are both on page 2 of the exam, they are the same version on each exam. Some of you had an exam that was all Version 1 [2], but most of you had exams for which each version was represented. This process is fair if you agree with me that, for every question, the versions are of almost identical difficulty.

1. **Problem 1: 3 points.**

The smaller P-value is 0.2454 [0.2978] on Version 1 [2]. For both versions, let $B$ denote the P-value for $>$; let $C$ denote the P-value for $<$; and let $D$ denote the P-value for $\neq$.

**Version 1:**
The first thing to note is that 
\[ x = \hat{p}_1 - \hat{p}_2 = 0.35 - 0.30 = 0.05, \]
is a positive number. This implies that 
\[ B \leq 0.5000 < C, \]
and that 
\[ D = 2B. \]

Also, recall that $B + C$ must be greater than 1. We now argue by contradiction.

- I start by determining which P-value is 0.8326. This cannot be $B$ because it exceeds 0.5000.

If $D = 0.8326$, then 
\[ B = D/2 = 0.4163. \]

By the process of elimination, 
\[ C = 0.2454. \]
Thus, $B + C < 1$, which is impossible.

**Version 2:**
The first thing to note is that 
\[ x = \hat{p}_1 - \hat{p}_2 = 0.25 - 0.30 = -0.05, \]
is a negative number. This implies that 
\[ C \leq 0.5000 < B, \]
and that 
\[ D = 2C. \]

Also, recall that $B + C$ must be greater than 1. We now argue by contradiction.

- I start by determining which P-value is 0.8119. This cannot be $C$ because it exceeds 0.5000.
If $D = 0.8119$, then 

$$C = D/2 = 0.4059.$$ 

By the process of elimination, 

$$B = 0.2978.$$ 

Thus, $B + C < 1$, which is impossible. 

Thus, $B = 0.8119$. 

- If $D = 0.2978$, then 

$$C = D/2 = 0.1489.$$ 

From above, 

$$B = 0.8119.$$ 

Thus, $B + C < 1$, which is impossible. 

Thus, $B = 0.8119$. 

2. **Problem 2: 3 points.**

**Version 1:**

The first thing to note is that 

$$x = \hat{p}_1 - \hat{p}_2 = 0.52 - 0.60 = -0.08,$$ 

is a negative number. Define the following numbers: 

$$B = P(X \leq -0.08); C = P(X \geq -0.08);$$ 

and 

$$D = P(X \geq +0.08).$$ 

(It might help to draw a picture on the number line.) Note that because we cannot assume symmetry, $B$ and $D$ need not be equal. We do know, however, that $B + C > 1$ and $C > D$. With this notation, the P-values are given in the table below.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&gt;$</td>
<td>$C$</td>
</tr>
<tr>
<td>$&lt;$</td>
<td>$B$</td>
</tr>
<tr>
<td>$\neq$</td>
<td>$B + D$</td>
</tr>
</tbody>
</table>

Let’s consider the P-value 0.2931.

- Suppose that $C = 0.2931$. Then 

$$B + C > 1 \text{ implies } B > 0.7069$$ 

and $B + D$ must equal 0.5242, which is impossible because it is smaller than $B$. 

- Suppose that $B + D = 0.2931$. Then 

$$B \leq 0.2931, \text{ making } C = 0.5242$$ 

and $B + C < 1$. 

Thus, $B = 0.2931$. $C$ cannot equal 0.5242 because this would make $B + C < 1$. Thus, $B + D = 0.5242$ and $C$ is unknown.

**Version 2:**

The first thing to note is that 

$$x = \hat{p}_1 - \hat{p}_2 = 0.69 - 0.60 = 0.09,$$ 

is a positive number. Define the following numbers: 

$$B = P(X \geq 0.09); C = P(X \leq 0.09);$$ 

and 

$$D = P(X \leq -0.09).$$ 

(It might help to draw a picture on the number line.) Note that because we cannot assume symmetry, $B$ and $D$ need not be equal. We do know, however, that $B + C > 1$ and $C > D$. With this notation, the P-values are given in the table below.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$&gt;$</td>
<td>$B$</td>
</tr>
<tr>
<td>$&lt;$</td>
<td>$C$</td>
</tr>
<tr>
<td>$\neq$</td>
<td>$B + D$</td>
</tr>
</tbody>
</table>

Let’s consider the P-value 0.1801.
• Suppose that $C = 0.1801$. Then $B + C > 1$ implies $B > 0.8199$
and $B + D$ must equal 0.2795, which is impossible because it is smaller than $B$.

• Suppose that $B + D = 0.1801$. Then $B \leq 0.1801$, making $C = 0.5242$
and $B + C < 1$.

Thus, $B = 0.1801$. $C$ cannot equal 0.2795 because this would make $B + C < 1$. Thus, $B + D = 0.2795$ and $C$ is unknown.

3. **Problem 3: 6 points.**

The box contains six [seven] cards for Version 1 [2]. The class did very well on this problem; hence, my solution below is rather terse.

**Version 1:**

(a) The completed table is below, with the fractions not reduced.

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>1</th>
<th>3</th>
<th>5.5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3/30</td>
<td>2/30</td>
<td>5/30</td>
</tr>
<tr>
<td>3</td>
<td>3/30</td>
<td>6/30</td>
<td>6/30</td>
<td>15/30</td>
</tr>
<tr>
<td>5.5</td>
<td>2/30</td>
<td>6/30</td>
<td>2/30</td>
<td>10/30</td>
</tr>
<tr>
<td>Total</td>
<td>5/30</td>
<td>15/30</td>
<td>10/30</td>
<td>1</td>
</tr>
</tbody>
</table>

For example, the probability of the cell (3,3) is $3/6$ (the probability the first card will be a ‘3’) multiplied by $2/5$ (the probability the second card will be a ‘3’ after the first card has been a ‘3’).

(b) First, we note that for the sum to equal 8.5, either cell (2,7) or (7,2) must occur. Reading from the table above, this probability is

\[ \frac{6}{42} + \frac{6}{42} = \frac{12}{42}. \]

(c) First, we note that for the cards to be equal, either cell (1,1), (3,3) or (5.5, 5.5) must occur. Reading from the table above, this probability is

\[ \frac{6}{30} + \frac{2}{30} = \frac{8}{30}. \]

**Version 2:**

(a) The completed table is below, with the fractions not reduced.

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6/42</td>
</tr>
<tr>
<td>3</td>
<td>3/42</td>
</tr>
<tr>
<td>5</td>
<td>9/42</td>
</tr>
<tr>
<td>7</td>
<td>3/42</td>
</tr>
<tr>
<td>Total</td>
<td>18/42</td>
</tr>
</tbody>
</table>

For example, the probability of the cell (5,5) is $3/7$ (the probability the first card will be a ‘5’) multiplied by $2/6$ (the probability the second card will be a ‘5’ after the first card has been a ‘5’).

(b) First, we note that for the sum to equal 9, either cell (2,7) or (7,2) must occur. Reading from the table above, this probability is

\[ \frac{3}{42} + \frac{3}{42} = \frac{6}{42}. \]

(c) First, we note that for the cards to be equal, either cell (2,2), (5,5) or (7,7) must occur. Reading from the table above, this probability is

\[ \frac{6}{42} + \frac{6}{42} + 0 = \frac{12}{42}. \]

4. **Problem 4: 4 points.**

The number of possible assignments is 36 [35] on Version 1 [2].

**Version 1:**

(a) Both critical regions have the $\geq$ sign; thus, the alternative for both tests is $>$.  

(b) This is simply

\[ P(U \geq 13) = \frac{3}{36} + \frac{5}{36} = \frac{8}{36}. \]

(c) This is simply

\[ P(R_1 \geq 13) = \frac{4}{36} + \frac{5}{36} = \frac{9}{36}. \]
(d) Either both tests reject or both tests fail to reject. Thus, the probability is
\[ \frac{24}{36} + \frac{5}{36} = \frac{29}{36}. \]

**Version 2:**

(a) Both critical regions have the \( \geq \) sign; thus, the alternative for both tests is \( > \).

(b) This is simply
\[ P(U \geq 19) = \frac{3}{35} + \frac{3}{35} = \frac{6}{35}. \]

(c) This is simply
\[ P(R_1 \geq 16) = \frac{1}{35} + \frac{3}{35} = \frac{4}{35}. \]

(d) Either both tests reject or both tests fail to reject. Thus, the probability is
\[ \frac{28}{35} + \frac{3}{35} = \frac{31}{35}. \]

5. **Problem 5: 3 points.**

Both versions have the same four intervals, but their labels have been changed. Thus, in the solution below I will refer to each interval by its lower bound, suppressing the decimal point for (my) ease; i.e., they are 519, 528, 536 and 544.

First, we calculate the four centers. We get 0.631 twice and two other numbers. The two approximate intervals must have the same center \( \hat{p} \); thus, 528 and 544 are the approximate intervals.

Interval 544 is narrower than interval 528; thus, the former is 90% and the latter is 95%.

Finally, 519 and 536 are the exact intervals. The former is wider; thus, it is 95% and the latter is 90%.

6. **Problem 6: 9 points.**

The number of runs in the simulation experiment is 2,500 [3,000] in Version 1 [2].

**Version 1:**

(a) The cell count described is ‘b;’ thus, the count is 100.

(b) The cell count described is ‘d;’ thus, the count is 700.

(c) This one is a bit tricky. Even though this is a question about power, it is a Chapter 12 question. The members of the population are the possible assignment and each assignment is a success if, and only if, the test rejects.

The data are \( n = 2,500 \) Bernoulli trials with unknown probability of success \( r_U \) and a total of 900 successes in the data. In the notation of Chapter 12, \( \hat{p} = \frac{900}{2500} = 0.360 \) and the 90% confidence interval is
\[ 0.360 \pm 1.645 \sqrt{\frac{0.36(0.64)}{2500}} = 0.360 \pm 0.016. \]

(d) The nearly certain interval is
\[ \frac{200 - 100}{2500} \pm (3/2500)\sqrt{100 + 200} = 0.040 \pm 0.021. \]

**Version 2:**

(a) The cell count described is ‘c;’ thus, the count is 150.

(b) The cell count described is ‘a;’ thus, the count is 1,500.

(c) This one is a bit tricky. Even though this is a question about power, it is a Chapter 12 question. The members of the population are the possible assignment and each assignment is a success if, and only if, the test rejects.

The data are \( n = 3,000 \) Bernoulli trials with unknown probability of success \( r_R \) and a total of 1,350 successes in the data. In the notation of Chapter 12, \( \hat{p} = \frac{1350}{3000} = 0.450 \) and the 90% confidence interval is
\[ 0.450 \pm 1.645 \sqrt{\frac{0.45(0.55)}{3000}} = 0.450 \pm 0.015. \]
(d) The nearly certain interval is
\[
\frac{300 - 150}{3000} \pm (3/3000)\sqrt{300 + 150} = 0.050 \pm 0.021.
\]

7. **Problem 7: 5 points.**
The value of \( x \) in part (a) is 3 [6] in Vers-
ion 1 [2].

**Version 1:**
(a) Reading from the table, the CI is
[0.108, 0.566].
(b) The CI contains \( p = 0.600 \) if, and only if, \( x \) equals 4, 5, 6, 7 or 8.
(c) The lower bound of the CI exceeds \( p = 0.300 \) if, and only if, \( x \) equals 6, 7, 8, 9 or 10.
(d) The upper bound of the CI is smaller than \( p = 0.700 \) if, and only if, \( x \) equals 0, 1, 2, 3 or 4.

**Version 2:**
(a) Reading from the table, the CI is
[0.341, 0.822].
(b) The CI contains \( p = 0.300 \) if, and only if, \( x \) equals 1, 2, 3, 4 or 5.
(c) The lower bound of the CI exceeds \( p = 0.500 \) if, and only if, \( x \) equals 8, 9 or 10.
(d) The upper bound of the CI is smaller than \( p = 0.800 \) if, and only if, \( x \) equals 0, 1, 2, 3, 4 or 5.

8. **Problem 8: 5 points.**
The total number of successes is 840 [1,340] in Version 1 [2].

**Version 1:**
It is important to go back in time before data are collected to set up the problem. We plan

to observe \( X \) which has a Poisson distribution with parameter \( \theta \). For later in the problem, we note that
\[
\theta = 15\lambda,
\]
where \( \lambda \) is the rate per hour.
The data yield \( X = 840 \), making the 95% CI for \( \theta \):
\[
840 \pm 1.96\sqrt{840} = 840 \pm 56.81.
\]
Next, we divide thru by 15 to convert to \( \lambda \), obtaining:
\[
56.00 \pm 3.79.
\]

**Version 2:**
It is important to go back in time before data are collected to set up the problem. We plan

to observe \( X \) which has a Poisson distribution with parameter \( \theta \). For later in the problem, we note that
\[
\theta = 20\lambda,
\]
where \( \lambda \) is the rate per hour.
The data yield \( X = 1340 \), making the 98% CI for \( \theta \):
\[
1340 \pm 2.326\sqrt{1340} = 1340 \pm 85.15.
\]
Next, we divide thru by 20 to convert to \( \lambda \), obtaining:
\[
67.00 \pm 4.26.
\]

9. **Problem 9: 4 points.**

**Version 1:**
First, we calculate \( r' = 12/15 = 0.8 \). The 90% PI is:
\[
0.8(840) \pm 1.645\sqrt{0.8(840)(1.8)} = 672 \pm 57.21.
\]

**Version 2:**
First, we calculate \( r' = 14/20 = 0.7 \). The 95% PI is:
\[
0.7(1340) \pm 1.96\sqrt{0.7(1340)(1.7)} = 938 \pm 78.3.
\]
10. **Problem 10: 4 points.**
The confidence level is 88% [92%] in Version 1 [2].

**Version 1:**
As with problem 8, it helps to go back in time to before the data are collected. Vince plans to observe $X$ which has a Poisson distribution with parameter $\theta$, where

$$\theta = 3\lambda,$$

where $\lambda$ is the rate per minute.

Vince observes $X = 4$ and the website gives Vince an upper bound, call it $b$, for $\theta$:

$$\theta \leq b \text{ or } \lambda \leq b/3.$$  

We are given that

$$b/3 = 2.5574; \text{ thus, } b = 7.6722.$$  

The inequality

$$\theta \leq 7.6722 \text{ becomes } 700p \leq 7.6722,$$

which gives

$$p \leq 0.01096.$$  

**Version 2:**
As with problem 8, it helps to go back in time to before the data are collected. Vince plans to observe $X$ which has a Poisson distribution with parameter $\theta$, where

$$\theta = 2\lambda,$$

where $\lambda$ is the rate per minute.

Vince observes $X = 3$ and the website gives Vince an upper bound, call it $b$, for $\theta$:

$$\theta \leq b \text{ or } \lambda \leq b/2.$$  

We are given that

$$b/2 = 3.5171; \text{ thus, } b = 7.0342.$$  

The inequality

$$\theta \leq 7.0342 \text{ becomes } 500p \leq 7.0342,$$

which gives

$$p \leq 0.014068.$$  

11. **Problem 11: 4 points.**

**Version 1:**
(a) You are given the sequence 111100. The other four sequences are:

$$111101, 011110, 101111 \text{ and } 001111.$$  

(b) There are five sequences that yield ($V = 4$). Thus, $P(V = 4)$ is the sum of the probabilities of these five sequences:

$$3p^4q^2 + 2p^5q.$$  

**Version 2:**
(a) You are given the sequence 1111100. The other four sequences are:

$$111101, 0111101, 1011111 \text{ and } 0011111.$$  

(b) There are five sequences that yield ($V = 5$). Thus, $P(V = 5)$ is the sum of the probabilities of these five sequences:

$$3p^5q^2 + 2p^6q.$$