Detailed Comments on Quizzes

June 28, 2014

1 Quiz 1

Every question on every quiz will have multiple versions (either two or three). On Quiz 1, each of the four questions has two versions, giving a total of

\[ 2 \times 2 \times 2 \times 2 = 16 \]

possible versions of the quiz.

1.1 Question 1

The histogram is symmetric about C and the width of each class interval is D. The two versions have different values of both C and D. The value of C is irrelevant; C is the center of gravity of the histogram and tells us nothing about spread.

It is impossible to compute the value of the SD from the information given. It is impossible to look at a histogram and know exactly the value of the SD. Thus, you need to find a different approach. First, note that I tell you that one of the numbers is the SD; thus, you just need to eliminate all but one possibility.

The only property we know about the SD is the empirical rule, which should work well because the histogram is bell-shaped. The empirical rule states that approx. 68% of the data will lie within one SD of the mean.

1. Suppose that \( SD = D \). This would imply that the two central rectangles contain about 68% of the data. Even a brief look at the histogram shows this is wrong; much less than one-half of the data lie in these intervals. Thus, any value \( \leq D \) cannot be the SD.

2. Suppose that \( SD = 4D \). This would imply that the eight central rectangles contain about 68% of the data. This is patently false; the intervals contain almost all of the data. Thus, \( 4D \) or any larger value in the list cannot be the SD.

3. All that remains is \( SD = 2D \). This looks reasonable; the four central rectangles appear to include about 68% of the data.

By the way, I gave one-half credit for the answer \( SD = D \), a very popular wrong answer.
1.2 **Question 2**

In the preamble you are told the value of $n$. You are given the frequency and asked to determine the height of the rectangle. One twist is that the frequency changes from part to part. You need to remember the following definitions.

1. For a frequency histogram, the height of a rectangle equals the frequency of the class interval.
2. For a relative frequency histogram, the height of a rectangle equals the relative frequency of the class interval.
3. For a density histogram, the height of a rectangle equals the relative frequency of the class interval divided by the width of the class interval.

1.3 **Question 3**

In this problem you are given $n$ and the height of the rectangle, which is smaller than one for both versions. You are also given the interval boundaries, from which you can determine the numerical value of $w$.

You are asked to determine how many observations fall in the interval.

For a frequency histogram, the height equals the frequency. A frequency cannot be a noninteger; thus, it is impossible for the picture to be a frequency histogram.

For a relative frequency histogram, the frequency (how many) equals the product of the height and $n$.

For a density histogram, the relative frequency equals the area of the rectangle; thus, the frequency equals the product of the area and $n$.

1.4 **Question 4**

Both versions have $n = 20$. This question requires you to think abstractly; to consider all possibilities.

If all 20 numbers are identical, then weird things can happen. For example, if all 20 numbers are equal to, say, 33, then no matter how many (fewer than 20) and which numbers you discard, the mean and median will never change from 33.

My first trick is that because the mean and median are different, then the 20 numbers cannot be identical. Thus, the following statements are true:

1. If you discard the smallest [largest] number, then the mean must increase [decrease].
2. If you discard both the smallest and largest numbers, then the mean could decrease, stay the same or increase.
3. The median is about positions. The median of the original data set is the average of the numbers in positions 10 and 11, which I will call original positions 10 and 11.
If you discard both the smallest and largest numbers, then original positions 10 and 11 are the center positions of the new data set and, hence, the median does not change.

If one value is discarded, then the situation is a bit trickier. If the smallest [largest] value is discarded, then the median becomes the number in the original position 11 [10]. If the same number lives in original positions 10 and 11, then the median does not change. If, however, the number in original position 10 is smaller than the number in original position 11, then the median will change in the obvious way: become larger [smaller] if the smallest [largest] value is discarded.

### 2 Quiz 2

There are five questions on Quiz 2, with two versions of each question.

#### 2.1 Question 1

For both versions, in part (a) subjects 1, 2, 4 and 6 are assigned to treatment 1. Thus, the assignment is 1,2,4,6.

**Version 1:**

1. For part (b), you are asked to calculate

   \[ \bar{x} = \frac{68}{4} = 17. \]

2. For part (c), you are asked to find \( \bar{x} \) for assignment 1,2,3,4; it is \( \frac{40}{4} = 10 \).

3. For part (d), you are asked to find \( \bar{y} \) for assignment 1,2,3,4; it is \( \frac{44}{2} = 22 \).

**Version 2:**

1. For part (b), you are asked to calculate

   \[ \bar{y} = \frac{16}{2} = 8. \]

2. For part (c), you are asked to find \( \bar{x} \) for assignment 3,4,5,6; it is \( \frac{52}{4} = 13 \).

3. For part (d), you are asked to find \( \bar{y} \) for assignment 3,4,5,6; it is \( \frac{32}{2} = 16 \).

#### 2.2 Question 2

If you understand how to use the randomizer website, this question was pretty easy, with one twist. The twist was that the subjects were not labeled

\[ 1, 2, 3, \ldots, n, \]
as is the norm.

**Version 1:** This version had $n_1 = 11$.
The answers are: 8, 11, 51, 70 and yes.

**Version 2:** This version had $n_1 = 17$.
The answers are: 9, 17, 61, 90 and yes.

### 2.3 Question 3

**Version 1:**

1. To obtain $P(U \leq -1.9)$, sum the probabilities at
$$-4.7, \ldots, -1.9$$ and obtain $6/21$.

2. Sum the probabilities at
$$-4.7, \ldots, -2.6$$ and $3.0, \ldots, 5.1$ and obtain $8/21$.

3. Sum the probabilities at
$$2.3, \ldots, 3.7$$ and obtain $4/21$.

**Version 2:**

1. To obtain $P(U \leq -3.3)$, sum the probabilities at
$$-4.7, \ldots, -3.3$$ and obtain $3/21$.

2. Sum the probabilities at
$$-4.7, -4.0, 3.7$$ and $5.1$ and obtain $4/21$.

3. Sum the probabilities at
$$3.0$$ and $3.7$ and obtain $3/21$.

### 2.4 Question 4

**Version 1:** This version has 12,000 runs.
The answers are: $2400/12000 = 0.2$; $3$; $0.2$; and 12,000.

**Version 2:** This version has 9000 runs.
The answers are: $2700/9000 = 0.3$; $3$; $0.3$; and 9000.

### 2.5 Question 5

**Version 1:** Assignments 1,4,5; 2,4,5; and 3,4,5 all put Mia and Anna on treatment 1; assignment 1,2,3 puts them on treatment 2.

**Version 2:** Assignments 3,4; 3,5; and 4,5; all put Al and Harrison on treatment 2; assignment 1,2 puts them on treatment 1.
3 Quiz 3

There are four questions on Quiz 3, with two versions of each question.

3.1 Question 1

Both versions give the same data from a balanced CRD on six subjects, which is reproduced below for your convenience.

<table>
<thead>
<tr>
<th>Unit:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response on Treatment 1:</td>
<td>a</td>
<td>18</td>
<td>c</td>
<td>24</td>
<td>e</td>
<td>6</td>
</tr>
<tr>
<td>Response on Treatment 2:</td>
<td>9</td>
<td>b</td>
<td>3</td>
<td>d</td>
<td>12</td>
<td>f</td>
</tr>
</tbody>
</table>

Each version asks three questions; I answer all six below even though only three are on any particular quiz.

- The Skeptic being correct implies: $c = 3$; and $d = 24$.
- A constant treatment effect (cte) of $+4$ implies: $18 - b = 4$, which gives $b = 14$.
- A cte of $+5$ implies: $a - 9 = 5$, which gives $a = 14$.
- A cte of $-3$ implies: $e - 12 = -3$, which gives $e = 9$.
- A cte of $-2$ implies: $6 - f = -2$, which gives $f = 8$.

3.2 Question 2

Both versions give the same sampling distribution and ask for three P-values. The sampling distribution is reproduced below for your convenience.

<table>
<thead>
<tr>
<th>$u$</th>
<th>$P(U = u)$</th>
<th>$u$</th>
<th>$P(U = u)$</th>
<th>$u$</th>
<th>$P(U = u)$</th>
<th>$u$</th>
<th>$P(U = u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.7</td>
<td>1/21</td>
<td>-1.9</td>
<td>2/21</td>
<td>0.9</td>
<td>3/21</td>
<td>3.0</td>
<td>2/21</td>
</tr>
<tr>
<td>-4.0</td>
<td>1/21</td>
<td>-1.2</td>
<td>2/21</td>
<td>1.6</td>
<td>1/21</td>
<td>3.7</td>
<td>1/21</td>
</tr>
<tr>
<td>-3.3</td>
<td>1/21</td>
<td>-0.5</td>
<td>2/21</td>
<td>2.3</td>
<td>1/21</td>
<td>5.1</td>
<td>1/21</td>
</tr>
<tr>
<td>-2.6</td>
<td>1/21</td>
<td>0.2</td>
<td>2/21</td>
<td></td>
<td></td>
<td>Total: 21/21</td>
<td></td>
</tr>
</tbody>
</table>

Version 1:

- The alternative is $>$ and $u = 1.6$. Thus, the P-value is $P(U \geq 1.6) = 6/21 = 2/7$.
- The alternative is $<$ and $u = -3.3$. Thus, the P-value is $P(U \leq -3.3) = 3/21 = 1/7$.
- The alternative is $\neq$ and $u = -3.3$. Thus, the P-value is $P(U \leq -3.3) + P(U \geq +3.3) = 3/21 + 2/21 = 5/21$. 
Version 2:

- The alternative is < and \( u = -1.2 \). Thus, the P-value is \( P(U \leq -1.2) = 8/21 \).
- The alternative is > and \( u = 2.3 \). Thus, the P-value is \( P(U \geq 2.3) = 5/21 \).
- The alternative is \( \neq \) and \( u = 2.3 \). Thus, the P-value is

\[
P(U \geq 2.3) + P(U \leq -2.3) = 5/21 + 4/21 = 9/21 = 3/7.
\]

### 3.3 Question 3

You are asked to interpret the results of a vassarstats simulation. You first must note whether \( u \) is a positive number, a negative number or 0. In both versions, \( \bar{x} < \bar{y} \), making \( u < 0 \). Thus, the one-tailed P-value given by vassarstats is for the alternative <.

1. The alt. is >. Vassarstats does not provide this P-value. Thus, the P-value is unknown. It also is correct to say that it is larger than 0.5000.
2. The alt. is <. Thus, the P-value is the number given by one-tailed.
3. The alt is \( \neq \). Thus, the P-value is the number given by two-tailed.
4. Replace \( \hat{r} \) by the number given by two-tailed and \( m \) by 10,000.

### 3.4 Question 4

This question is similar to question 3, but for both versions, \( u = 0 \). This means that the exact P-value for \( \neq \) is 1; i.e., we don’t need to simulate. It also means, sadly, that vassarstats is useless for alternative > or <; it gives nonsense.

We could say the following, although I don’t expect you to do this. For the alternative >, the exact P-value is \( P(U \geq 0) \). For the alternative <, the exact P-value is \( P(U \leq 0) \). By symmetry these two P-values are equal. Their sum is greater than 1. (It equals, in fact, \( 1 + P(U = 0) \). Thus, it would be correct to say that the P-value for > is larger than 0.5000, but I would give full credit to the answer ‘unknown.’