Practice Exam Questions and Solutions
for Midterm 1
Statistics 301, Professor Wardrop

Chapter 5

1. On each of five days next week (Monday thru Friday), Earl will shoot six free throws. Assume that Earl's shots satisfy the assumptions of Bernoulli trials with $p = 0.37$.

   (a) Compute the probability that on any particular day Earl makes his first free throw and misses his second and third free throws.

   (b) Compute the probability that on any particular day Earl obtains a total of exactly two successes. For future reference, if Earl obtains exactly two successes on any particular day, then we say that the event "Brad" has occurred.

   (c) Refer to part (b). Compute the probability that: next week Brad will occur exactly twice, with one of the occurrences on Tuesday. (Note: You are being asked to compute one probability.)

2. Below are tables (1–3) from three different studies. Match each Table to the correct statement below.

   (a) This table provides evidence that p increased over the course of the study.

   (b) This table provides evidence that p decreased over the course of the study.

   (c) This table provides no evidence that p changed over the course of the study.

Table 1

Half	S	F	Total
1st	30	45	75
2nd	40	35	75

Table 2

Half	S	F	Total
1st	60	90	150
2nd	60	90	150

Table 3

Half	S	F	Total
1st	90	110	200
2nd	50	150	200

Table 4

Prev. \ Current	S	F	Total
S	60	59	119
F	60	80	140
Total	120	139	259

Table 5

Prev. \ Current	S	F	Total
S	70	50	120
F	49	30	79
Total	119	80	199

Table 6

Prev. \ Current	S	F	Total
S	18	42	60
F	42	98	140
Total	60	140	200

Above are tables (4, 5 and 6) from three different studies. Note that at least one of these tables is the answer to more than of the following four questions.

(d) Which table is for the study that had an F on its first trial and an S on its last trial?

(e) Which table is for the study in which the subject performed better (S's are good) after an S than after an F?

(f) Which table is for the study that had an F on its first and last trials?

(g) Which table is for the study in which the subject performed the same after an S as after an F?

Chapter 6

3. A random sample of size $n = 452$ yields 113 successes. Calculate the 95% confidence interval for p.

4. George enjoys throwing horse shoes. Last week he tossed 150 shoes and obtained 36 ringers. (Ringers are good.) Next week he plans to throw 250 shoes. Assume that George's tosses satisfy the assumptions of Bernoulli trials.

   (a) Calculate the point prediction of the number of ringers that George will obtain next week.

   (b) Calculate the 90% prediction interval for the number of ringers George will obtain next week.

   (c) It turns out that next week George obtains 62 ringers. Given this information, comment on your answers in parts (a) and (b).

5. Carl selects one random sample from a population and calculates three confidence intervals for p. His intervals are below.

A	B	C
$\hat{p} \pm 0.080$	$\hat{p} \pm 0.040$	$\hat{p} \pm 0.072$

   Match each confidence interval to its level, with levels chosen from: 80%, 90%, 95%, 98%, and 99%. Note: Clearly, two of these levels will not be used.

6. Fiona selects a random sample and calculates five confidence intervals for p; i.e. all five intervals are computed from the same data. Fiona **does not** restrict herself to the five standard confidence levels we use. Below are her five intervals.

Label:	A	B
CI:	$[0.594, 0.666]$	$[0.601, 0.679]$
C	D	E
$[0.620, 0.660]$	$[0.580, 0.680]$	$[0.555, 0.705]$

   Given that exactly two of the CI's contain reflect calculation errors, which three CI's are calculated correctly? Do not provide your reason(s).

Chapter 7

7. An observational study yields the following "collapsed table."

Group	S	F	Total
1	72	228	300
2	88	212	300
Total	160	440	600

   Below are two component tables for these data. Complete these tables so that Simpson's Paradox is occurring **or** explain why Simpson's Paradox *cannot* occur for these data. For the latter, you must provide computations that justify your answer.

	Subgp A				Subgp B		
Gp	S	F	Tot	Gp	S	F	Tot
1	30		60	1	42		240
2			120	2			180
Tot			180	Tot			420

8. An observational study yields the following collapsed table.

Group	S	F	Total
1	41	59	100
2	35	65	100
Total	76	124	200

   Below are two component tables for these data. Complete these tables so that Simpson's Paradox is occurring **or** explain why Simpson's Paradox *cannot* occur for these data. For the latter, you must provide computations that justify your answer.

	Subgp A				Subgp B		
Gp	S	F	Tot	Gp	S	F	Tot
1	5		20	1	36		80
2			40	2			60
Tot			60	Tot			140

9. In a 'Chapter 7' problem, you are given the following information:

$$\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}} = 0.025, \text{ and}$$

$$\frac{\hat{p}_2\hat{q}_2}{n_2} = 0.005.$$

Calculate the half-width of the 90% confidence interval for $p_1 - p_2$.

Solutions

1. (a) This is a multiplication rule problem b/c it is a question about a particular sequence.

$$P(SFF) = pqq = 0.1469.$$

(b) This is a binomial problem b/c it is about the total number of successes. The probability of exactly two successes is

$$\frac{6!}{2!4!}(0.37)^2(0.63)^4 =$$

$$15(0.1369)(0.1575) = 0.3234.$$

(c) For this part each day is a trial and the probability that a day yields a success is, from part (b), $p = 0.3234$. Then it gets trickier. Let Y denote the total number of successes (Brads) Earl gets on Monday, Wednesday, Thursday and Friday. We want the probability of a Brad on Tuesday **and** $(Y = 1)$. These probabilities are $p = 0.3234$ and

$$\frac{4!}{1!3!}(0.3234)(0.6766)^3 = 0.4007.$$

Finally, by the multiplication rule, we multiply these answers to get

$$0.3234(0.4007) = 0.1296.$$

2. (a) This describes Table 1.

(b) This describes Table 3.

(c) This describes Table 2.

(d) There will be one subtracted from the column total for F and one subtracted from the row total for S. Thus, the row total for F will be one larger than the column total for F; only Table 4 satisfies this condition.

(e) We need $\hat{p}_1 > \hat{p}_2$; only Table 4 satisfies this condition.

(f) There will be one subtracted from the column total for F and one subtracted from the row total for F. Thus, the row and column totals for F will be the same number; only Table 6 satisfies this condition.

(g) We need $\hat{p}_1 = \hat{p}_2$; only Table 6 satisfies this condition.

3. First, $\hat{p} = 113/452 = 0.250$. The 95% confidence interval is

$$0.250 \pm 1.96\sqrt{\frac{0.25(0.75)}{452}} =$$

$$0.250 \pm 0.040 = [0.210, 0.290].$$

4. (a) First, $\hat{p} = 36/150 = 0.24$. Thus, the point prediction is

$$m\hat{p} = 250(0.24) = 60.$$

(b) The 90% prediction interval is $60\pm$

$$1.645\sqrt{60(0.76)}\sqrt{1 + (250/150)} =$$

$$60 \pm 18.14 = [42, 78],$$

after rounding.

(c) With 62 ringers, the point prediction is too small by 2, but the prediction interval is correct b/c 62 is between 42 and 78.

5. Based on the half-widths of the CIs, B has the smallest confidence level and A has the largest. But here is the key. The CIs differ only in which z they use, namely three of the following: 1.282, 1.645, 1.96, 2.326 and 2.576. Note

that A is twice as wide as B; by inspection, this implies that A uses 2.576 and B uses 1.282 b/c these are the two z's such that one is twice as large as the other. Thus, A is 99% and B is 80%. Finally, $0.072/0.080 = 0.9$. Thus, the z for C is 90% of 2.576, i.e. $(0.9)(2.576) = 2.3184$, or, allowing for rounding error, $z = 2.326$ and C is 98%.

6. Intervals A, D and E are centered at 0.630, while B and C are centered at 0.640. But all five CI's should have the same center. A, D and E must be the correct CI's.

7. In the collapsed table, $\hat{p}_1 < \hat{p}_2$. To get a reversal, we need $c \leq 59$ in Subgroup A and $c \leq 31$ in Subgroup B. Also, the two c's must sum to the 88 in the collapsed table. There are three possible answers: 59 and 29; 58 and 30; 57 and 31.

8. In the collapsed table, $\hat{p}_1 > \hat{p}_2$. To get a reversal, we need $c \geq 11$ in Subgroup A and $c \geq 28$ in Subgroup B. Also, the two c's must sum to the 35 in the collapsed table. This combination of restrictions is impossible.

9. The half-width is

$$1.645\sqrt{(0.025)^2 + 0.005} = 0.123.$$