DISCUSSION 11

1 Review

- **Error Rates Associated with Accepting/Rejecting Null Hypothesis**
  
  With null distribution \( \bar{X} \sim N(\mu_0, \sigma/\sqrt{n}) \) and alternative distribution \( \bar{X} \sim N(\mu_1, \sigma/\sqrt{n}) \)
  
  1. Confidence level = \( P(\text{Accept } H_0 | H_0 \text{ is true}) = 1 - \alpha \)
  2. Significance level = \( P(\text{Reject } H_0 | H_0 \text{ is true}) = 1 - \alpha = \text{Type I error} \)
  3. \( P(\text{Accept } H_0 | H_0 \text{ is false}) = \beta = \text{Type II error} \)
  4. Power = \( P(\text{Reject } H_0 | H_0 \text{ is false}) = 1 - \beta \)

- **Point Estimation of the Binomial Parameter \( \pi \)**
  
  Let \( x \) be a binomial random variable with parameters \( n \) and \( \pi \). An unbiased estimator of \( \pi \) is given by the sample proportion of events \( \hat{\pi} \). Its standard error is given exactly by \( \sqrt{\hat{\pi}(1 - \hat{\pi})/n} \) and is estimated by \( \sqrt{\hat{\pi}(1 - \hat{\pi})/n} \).

- **Normal-Theory Method for Obtaining a CI for the Binomial Parameter \( \pi \)**
  
  An approximate 100\% \( \times (1 - \alpha) \) confidence interval for the binomial parameter \( \pi \) based on the normal approximation to the binomial distribution is given by
  
  \[ \hat{\pi} \pm z_{\alpha/2} \sqrt{\frac{\hat{\pi}(1 - \hat{\pi})}{n}} \]

  This method of interval estimation should only be used if \( n\hat{\pi}(1 - \hat{\pi}) \geq 5 \).

- **One-Sample Test for a Binomial Proportion-Normal-Theory Method (Two-Sided Alternative)**
  
  Let the test statistic \( z = (\hat{\pi} - \pi_0)/\sqrt{\pi_0(1 - \pi_0)/n} \).
  
  If \( z < -z_{\alpha/2} \) or \( z > z_{\alpha/2} \), then \( H_0 \) is rejected. Otherwise, \( H_0 \) is accepted.
  
  This test should only be used if \( n\hat{\pi}(1 - \hat{\pi}) \geq 5 \).

2 Problems

(Textbook)

1. Suppose a clinical trial is conducted to test the efficacy of a new drug, spectinomycin, for treating gonorrhea is females. Forty-six patients are given a 4-g daily dose of the drug and are seen one week later, at which time 6 of the patients still have gonorrhea.

6.35 What is the best point estimate for \( \pi \), the probability of a failure with the drug.

6.36 What is a 95\% CI for \( \pi \)?

2. The drug erythromycin has been proposed to possibly lower the risk of premature delivery. A related area of interest is its association with the incidence of side effects during pregnancy. Assume 30\% of all pregnant women complain of nausea between weeks 24 and 28 of pregnancy. Furthermore, suppose that of 200 women who are taking erythromycin regularly during the period, 110 complain of nausea.

7.23 Test the hypothesis that the incidence rate of nausea for the erythromycin group is the same for a typical pregnant woman.
6-3. Consider the distribution of serum cholesterol levels for all 20- to 74-year-old males living in the United States. The mean of this population is 211 mg/dL, and the standard deviation is 46.0 mg/dL. In a study of a subpopulation of such males who smoke and are hypertensive, it is assumed (not unreasonably) that the distribution of serum cholesterol levels is normally distributed, with unknown mean \( \mu \), but with the same standard deviation \( \sigma \) as the original population.

(a) Formulate the **null hypothesis** and complementary **alternative hypothesis**, for testing whether the unknown mean serum cholesterol level \( \mu \) of the subpopulation of hypertensive male smokers is equal to the known mean serum cholesterol level of 211 mg/dL of the general population of 20- to 74-year-old males.

(b) In the study, a random sample of size \( n = 12 \) hypertensive smokers was selected, and found to have a sample mean cholesterol level of \( \bar{x} = 217 \) mg/dL. Construct a 95% **confidence interval** for the true mean cholesterol level of this subpopulation.

(c) Calculate the **p-value** of this sample, at the \( \alpha = .05 \) significance level.

(d) Based on your answers in parts (b) and (c), is the null hypothesis rejected in favor of the alternative hypothesis, at the \( \alpha = .05 \) significance level? **Interpret your conclusion:** What exactly has been demonstrated, based on the empirical evidence?

(e) Determine the 95% **acceptance region** and complementary **rejection region** for the null hypothesis. Is this consistent with your findings in part (d)? Why?