2.(a) i) A Venn diagram would be useful, can find that $P(A \cap B') = 0.25$ and $P(A' \cap B) = 0.15$; so $P(\text{exactly 1 type of defect}) = P(A \cap B') + P(A' \cap B) = 0.25 + 0.15 = 0.40$.

ii) Since $P(B) = 0.15 + 0.05 = 0.20$, $P(A|B) = P(A \cap B)/P(B) = 0.05/0.20 = 0.25$; since $P(A|B) \neq P(A) = 0.30$, A and B are not independent events.

(b) i) X = number that have positive reaction in 5 patients has a binomial distribution, with $n = 5$, $p = 2/5$, so

$$P(X = 3) = \frac{5!}{3!\,2!}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^2 = 10\,2^3\,3^2/5^5 = 0.2304$$

ii) $E(X) = np = 150\,(2/5) = 60$, St.Dev.$(X) = \sqrt{150\,(2/5)\,(3/5)} = 6$. Use normal approximation, so

$$P(X > 65) = P(X > 65.5) = P\left(\frac{X-60}{6} > \frac{65.5-60}{6}\right) = P(Z > 0.916) = 1 - \Phi(0.916) = 1 - 0.8204 = 0.1796$$

(c) i) $E(X) = np = 500(0.008) = 4$

ii) Use Poisson approximation, with $\lambda = E(X) = 4$, so
$P(X = 6) \approx e^{-4}4^6/6! = 0.1042 \equiv F(6;4) - F(5;4) = 0.889 - 0.785$, from Table 2.

3. Use that $Z = (X-29)/2.5$ is standard normal $N(0,1)$. Then, for instance, in (a),

$$P(X > 32) = P\left(\frac{X-29}{2.5} > \frac{32-29}{2.5}\right) = P(Z > 1.2) = 1 - P(Z < 1.2) = 1 - 0.8849 = 0.1151$$

(c) Want c so that $0.75 = P(X > c) = P(Z > (c-29)/2.5) \equiv P(Z > -z_{.25})$, where $z_{.25} = 0.675$ from Table 3. This implies that $(c-29)/2.5 = -0.675$, or $c = 29 - 2.5(0.675) = 27.3125$.

4. (a) X = number of defects in 6 sheets has Poisson distribution, with parameter $\lambda = (1/2)\,6 = 3$, so obtain $P(X > 5) = 1 - P(X \leq 5) = 1 - \sum_{x=0}^{5} e^{-3}3^x/x! = 1 - 0.916 = 0.084$, using Table 2.

(b) i) $P(X > 2) = \int_2^\infty \frac{1}{\beta}\,e^{-x/\beta}\,dx = -\,e^{-x/\beta}\big|_2^\infty = -\,e^{-\infty} + e^{-2/5} = e^{-.4} = 0.6703$

ii) Suppose we denote the independent events $A = (X_1 > 2)$ and $B = (X_2 > 2)$, then we want

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6703 + 0.6703 - P(A)\,P(B) = 2\,(0.6703) - (0.6703)^2 = 0.8913$$

This also equals $1 - P(A' \cap B') = 1 - (0.3297)^2 = 0.8913$. Can also relate in terms of binomial distribution, letting Y = number of bulbs out of 2 that last for at least two years.

5. (a) $\mu = \sum_{x=2}^{5} x\,f(x) = 2\,(1/8) + 3\,(1/4) + 4\,(1/4) + 5\,(3/8) = 31/8 = 3.875$. Similarly, can find $E(X^2) = \sum_{x=2}^{5} x^2\,f(x) = 4\,(1/8) + 9\,(1/4) + 16\,(1/4) + 25\,(3/8) = 129/8 = 16.125$. Thus, $\text{Var}(X) = \sigma^2 = E(X^2) - \mu^2 = (129/8) - (31/8)^2 = 71/64 = 1.109$.

(b) Since $X \equiv -\beta\ln(U) \leq x$ is equivalent to $U \geq e^{-x/\beta}$, as noted, and U has pdf $f_U(u) = 1$, $0 < u < 1$, the cdf of r.v. X is, by defintion,

$$G(x) = P(X \leq x) = P(U \geq e^{-x/\beta}) = \int_{e^{-x/\beta}}^{1} f_U(u)\,du = u\big|_{e^{-x/\beta}}^{1} = 1 - e^{-x/\beta}, \quad \text{for } x > 0$$

Thus the pdf of X is $g(x) = G'(x) = (1/\beta)\,e^{-x/\beta}$, for $x > 0$, which is the pdf of the exponential distribution.