1. Random-walk Metropolis. The Metropolis sampler refers to the original version of Metropolis et al. (1953). For proposal distribution, we assume \( q(y|x) = q(|y - x|) \). See example \( Y \sim N(x, 1) \) in lecture 20. In this case, 

\[
\alpha(X_i, Y) = \min\left( \frac{\pi(Y)}{\pi(X_i)}, 1 \right).
\]

Given \( X_i = x_i \),
1. Generate \( Y \sim g(|y - x_i|) \).
2. \( X_{i+1} = Y \) with probability \( \alpha(X_i, Y) \) and \( X_{i+1} = X_i \) otherwise.

Generating \( Y \sim g(y - x_i) \) is equivalent to \( Y = X_i + \epsilon_i \), where \( \epsilon_i \sim g(y) \). So we have random walk \( X_{i+1} = X_i + B_i \epsilon_i \) where \( B_i \) is Bernoulli with \( P(B = 1) = \alpha \).

Let’s generate \( N(0, 1) \) from \( Unif(-2, 2) \). 

```matlab
n=10000; x(1)=0; snrpdf=inline('exp(-x^2/2)'); for i=2:n 
    y=x(i-1) + 2*(1-2*rand); 
    alpha=min([1,snrpdf(y)/snrpdf(x(i-1))]); 
    u=rand; 
    if u <= alpha 
        x(i)=y; 
    else 
        x(i)=x(i-1); 
    end 
end
```

1. Generate \( Y \sim q(y) \).
2. \( X_{i+1} = Y \) with probability \( \alpha(X_i, Y) \) and \( X_{i+1} = X_i \) otherwise.

The independent sampler can be compared to Accept-reject method for generating random numbers. In AR method,
1. Generate \( Y \sim q(y) \).
2. \( X = Y \) with probability \( \frac{\pi(Y)}{q(Y)} \).

It can be shown that the independent Metropolis-Hastings algorithm is more efficient than the Accept-Reject method since independent M-H will accept more proposed values (see Robert and Casella, Monte Carlo Statistical Method for detail).

3. The Gibbs sampler is a special case of the Metropolis-Hastings algorithm but it differ in two ways:
   1. We always accept \( Y \) (candidate point).
   2. Need to know full conditional distributions.

By construction, the Gibbs sampler is multidimensional. Here we will only consider 2-dimensional case. Given joint density \( f(x, y) \), the Gibbs sampler is a way to generate the marginal density 

\[
f_X(x) = \int f(x, y) \, dy \quad \text{and} \quad f_Y(y) = \int f(x, y) \, dx.
\]

We will study the Gibbs sampler in lecture 23.