1. Go to www.infinityassociates.com to download MATLAB examples in the textbook.

2. Other factorization technique for covariance matrix $V$. Suppose $V$ has positive eigenvalues $\lambda_i$ and corresponding normalized eigenvectors $e_i$. Let $Q = (e_1, e_2, e_3)$. Then $QQ' = I$ and $V = Q\text{Diag}(\lambda_1, \lambda_2, \lambda_3)Q'$. So $V^{1/2} = Q\text{Diag}(\lambda_1^{1/2}, \lambda_2^{1/2}, \lambda_3^{1/2})Q'$. For $z \sim N(0, I)$, $w = V^{1/2}z \sim N(0, V)$.

3. Review of lecture 6. For given bivariate data $w_i = (x_i, y_i)$, we computed the sample covariance matrix $\hat{V}$. Then we estimated the Cholesky factor of $\hat{V}$. Based on this, $w = V^{1/2}z + \mu$ becomes $z = V^{-1/2}(w - \mu) \sim N(0, I)$. So we can test if $w_i$ are from bivariate normal by checking if $z_i \sim N(0, 1)$.

4. Consider linear equation

\[
\begin{pmatrix}
2 & 1 \\
4 & 2
\end{pmatrix}
\begin{pmatrix}
y_1 \\
y_2
\end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.
\]

There is no exact solution but it is possible to get a solution in the least-squares sense. Note that

\[
2y_1 + y_2 - 1 = \epsilon_1, \quad 4y_1 + y_2 - 1 = \epsilon_2.
\]

Minimize the sum of squared errors, $\min(X\beta - c)'(X\beta - c)$, where $c = (1, 1)'$, $\beta = (y_1, y_2)'$ and

\[
X = \begin{pmatrix}
2 & 1 \\
4 & 2
\end{pmatrix}, \quad \frac{\partial}{\partial \beta} \beta'A = A, \quad \frac{\partial}{\partial \beta} \beta'A\beta = 2A\beta.
\]

Then it can be shown that $\hat{\beta} = (X'X)^{-1}X'c$. 