Due Date: September 22, 1:20pm. No rate assignments will be accepted or graded. There will be 10% bonus points for exceptional solutions.

1. (Computational) If \( X \sim N(0, 1) \), the c.d.f. of \( X \) is given by

\[
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(\frac{-z^2}{2}\right) dz.
\]

It can be shown that

\[
\Phi^{-1}(x) \approx t - \frac{a_0 + a_1 t}{1 + b_1 t + b_2 t^2}
\]

for constants \( a_0 = 2.30753 \), \( a_1 = 0.27061 \), \( b_1 = 0.99229 \), \( b_2 = 0.04481 \) \((t^2 = -2 \log x)\) (Abramowitz and Stegun, Handbook of mathematical functions, 1964). Write a function that generate the standard normal random numbers using the integral transform method. Is your random number generator sufficiently good enough?

2. (Computational) Cauchy distribution is defined as \( g(x) = \frac{1}{1+x^2}, x \in \mathbb{R} \). Write a function that generate Cauchy random numbers. You may need to refer to a calculus textbook for the integral.

3. (Computational) Using the Cauchy random number generator, write a function that produce the standard normal random numbers using the accept-reject method. Compare the performance of the two standard normal random number generators using the Q-Q plot and other methods.

4. (Theoretical) When you generate the Q-Q plot of \( N(\mu_1, \sigma_1^2) \) and \( N(\mu_2, \sigma_2^2) \) for some arbitrary \( \mu_i \) and \( \sigma_i \), you will always get a straight line. Prove this fact by determining the equation of the line.