Stats 312: Lecture 9
Chi-squared distributions

Moo K. Chung
mchung@stat.wisc.edu
August 26, 2002

Concepts

1. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$.
\[
\frac{(n-1)S^2}{\sigma^2} = \frac{1}{\sigma^2} \sum_{j=1}^{n} (X_j - \bar{X})^2 \sim \chi^2_{n-1}
\]

2. Critical values for $\chi^2$:
\[
P(\chi^2_{1-\alpha/2,n} < \chi^2_n < \chi^2_{\alpha/2,n}) = 1 - \alpha
\]

3. 100(1 - $\alpha$)% CI for $\sigma^2$:
\[
\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}
\]

4. $X \sim \text{exp}(\lambda)$; exponential distribution with parameter $\lambda$. The density function is defined for $x \geq 0$.
\[
f(x) = \frac{1}{\lambda} e^{-x/\lambda}
\]

In-class problems

Continuing Example 7.12. Fat content of 10 randomly selected hot dogs. Assuming that observations come from normal distribution, find a 95% CI for the population variance of fat content.

> x<-c(25.2, 21.3, 22.8, 17.0, 29.8, 21.0, 25.5, 16.0, 20.9, 19.5)
> sd(x)
[1] 4.13414
> qchi.sq(0.025, 9)
[1] 1.902277
> qchi.sq(0.975, 9)
[1] 2.700389

Review problem

Lifetime of an electrical component has an exponential distribution with parameter $\lambda$. Estimate $\lambda$ by matching moments.

> x<-c(41.53, 18.73, 2.99, 30.34, 12.33, 117.52, 73.02, 223.63, 4.00, 26.78)
> mean(x)
[1] 55.087

Obtain the maximum likelihood estimate of the probability that lifetime exceeds 10.

> exp(-10/55.09)
[1] 0.8340006

Suppose that there are 100 electrical components with $\bar{x} = 55$, $s = 40$. Find a 95% confidence interval for $\lambda$.

> 55+qnorm(0.975)*40/10
[1] 62.83986
> 55-qnorm(0.025)*40/10
[1] 47.16014

Self-study problems

Example 7.15., Exercise 7.43.