Concepts

1. Least-squares estimator $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{x}$ is distributed as normal with mean $\beta_0$ and variance $\sigma^2 \bar{x}^2 / S_{xx}$.

2. Testing $H_0: \beta_0 = 0$ is based on $T = \frac{\hat{\beta}_0 - \beta_0}{S_{\hat{\beta}_0}} \sim t_{n-2}$, where $S_{\hat{\beta}_0} = \sigma^2 \bar{x}^2 / S_{xx}$.

3. Coefficient of determination measures the proportion of variability explained by the fitted model and it is defined as
   $$ r^2 = 1 - \frac{SSE}{SST} $$

4. Since $SST = S_{yy}$ and $SSE = S_{yy} - S_{xy}^2 / S_{xx}$, we can see that $r^2 = S_{xy}^2 / (S_{xx} S_{yy})$. $r$ is called the sample correlation coefficient and it will be studied in Lecture 21.

In-class problems

Example 1. Hooke’s law states that the length change $y$ in spring is proportional to applied force $x$, i.e. $y = \beta_1 x$

<table>
<thead>
<tr>
<th>x (kg)</th>
<th>29.4</th>
<th>39.2</th>
<th>49.0</th>
<th>58.8</th>
<th>68.6</th>
<th>78.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y (mm)</td>
<td>4.25</td>
<td>5.25</td>
<td>6.50</td>
<td>7.85</td>
<td>8.75</td>
<td>10.10</td>
</tr>
</tbody>
</table>

Fit the model $Y = \beta_0 + \beta_1 x + \epsilon$ and see if $\beta_0$ is in fact 0.

```r
> ra <- cov(y, y) / cov(x, x) - (cov(x, y) / cov(x, x))^2
> S0 <- sqrt(mean(x^2) / 4 * ra)
> b0 <- mean(y) - mean(x) * cov(x, y) / cov(x, x)
> b0 / S0
[1] 4.013
> 2 * pt(-4.013, 4)
[1] 0.016
```

```r
> x <- c(29.4, 39.2, 49.0, 58.8, 68.6, 78.4)
> y <- c(4.25, 5.25, 6.50, 7.85, 8.75, 10.10)
> summary(lm(y ~ x))

Coefficients: value Pr(>|t|)
(Intercept) .... 0.66 0.16 4.00 0.016
x .... 0.12 0.003 41.24 2.07e-06
Multiple R-Squared: 0.9977
```

Example 2. Suppose that data $(x_j, y_j)$ satisfy a circular relationship $Y_j = \sqrt{10^2 - x_j^2} + \epsilon_j$. This is a nonlinear relationship so we expect not to have a good linear fit.

```r
>x <- -10:10
>y <- sqrt(10^2 - x^2)
>summary(lm(y ~ x))

Coefficients: value Pr(>|t|)
(Intercept) .... 1.24e-09
x .... 4.47e-17
Multiple R-Squared: 1.378e-30
```

Self-study problems

Construct $100(1 - \alpha)\%$ confidence intervals for both $\beta_0$ and $\beta_1$. Show $\text{Var}(\hat{\beta}_0) = \sigma^2 / S_{xx}$. Example 12.9.