Stat 312: Lecture 13
Testing on Population Means

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1. Since we can minimize both type I and type II errors simultaneously in constructing a test procedure, we will control for fixed type I error (level α) and make type II error as small as possible. The corresponding test procedure is called a *level α test*.

2. Testing mean μ with known variance σ^2.

$$H_0 : \mu = \mu_0 \text{ vs. } H_1 : \mu < \mu_0$$

Test statistic: $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$, which is a pivot under H_0. A *pivot* is a statistic whose distribution is independent of population paramters. Rejection region for level α test: $z \leq -z_\alpha$.

Ex. Intelligence quotient (IQ) is a number used to express the relative intelligence of a person. An average person has the IQ of 100. Assume IQ of a dog follows $X_i \sim N(\mu, 10^2)$. The IQs of 10 dogs are measured: 30, 25, 70, 110, 40, 80, 50, 60, 100, 60. We want to test if dogs are as smart as people by testing

$$H_0 : \mu = 100 \text{ vs. } H_1 : \mu < 100$$

at level $\alpha = 0.05$.

```
> x<-c(30, 25, 70, 110, 40, 80, 50, 60,
100, 60)
> mean(x)
[1] 62.5
> z<-(mean(x)-100)/(10/sqrt(10))
> z
[1] -11.85854
> qnorm(0.05)
[1] -1.644854
> qnorm(0.025)
[1] -1.959964
```

Since $z < -z_{0.05} = -1.64$, we reject H_0 at $\alpha = 0.05$ level.

3. Testing mean μ with unknown variance σ^2.

$$H_0 : \mu = \mu_0 \text{ vs. } H_1 : \mu < \mu_0$$

Test statistic: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$. Rejection region for level α test: $z \leq -t_{\alpha,n-1}$.

Ex. IQ of a dog, $X_i \sim N(\mu, \sigma^2)$, where σ is unknown. Test

$$H_0 : \mu = 100 \text{ vs. } H_1 : \mu < 100$$

at level $\alpha = 0.05$.

```
> t=(mean(x)-100)/(sd(x)/sqrt(10))
> t
[1] -4.205156
> qt(0.05,9)
[1] -1.833113
```

A simpler method is to use command `t.test`.

```
>help(t.test)
...
t.test(x,alternative=c("two.sided", "less",
       "greater"),conf.level = 0.95)
...
> t.test(x,mu=100,alternative="less",
 conf.level=0.95)
        One Sample t-test
 data:  x t = -4.2036, df = 9,
 p-value = 0.001147
 alternative hypothesis: true mean is
 less than 100
 95 percent confidence interval:
    -Inf 78.8531
 sample estimates: mean of x 62.5
```

Review problems

Example 8.6., 8.7., 8.8., 8.9.