Stat 312: Lecture 10
Confidence intervals for variance

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1. Suppose the fat content of a hotdog follows normal distribution. 10 measurements are given.

```
> x<-c(25.2,21.3,22.8,17.0,29.8,21.0,
       25.5,16.0,20.9,19.5)
```

   We are interested in constructing interval estimate of the unknown population variance. To solve this problem, we need to know the following fact.

2. Let $X_1, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$.

$$\frac{(n-1)S^2}{\sigma^2} = \frac{1}{\sigma^2}\sum_{j=1}^{n}(X_j - \bar{X})^2 \sim \chi^2_{n-1}$$

   Quiz. What is the expectation of χ^2_{n-1}?

```
> y<- 0:50
> par(mfrow=c(2,2))
> plot(y,dchisq(y,1),type='l')
> plot(y,dchisq(y,5),type='l')
> plot(y,dchisq(y,10),type='l')
> plot(y,dchisq(y,20),type='l')
```

3. Critical values for χ^2_n distribution are defined as numbers that gives

$$P(\chi^2_{1-\alpha/2,n} < \chi^2_n < \chi^2_{\alpha/2,n}) = 1 - \alpha$$

   To find $\chi^2_{0.975,9}$ and $\chi^2_{0.025,9}$ that is need to construct 95% CI for σ^2, we use R package:

```
> qchisq(0.025,9)
[1] 2.700389
> qchisq(0.975,9)
[1] 19.02277
```

4. $100(1-\alpha)\%$ CI for σ^2:

$$\frac{(n-1)}{\chi^2_{\alpha/2,n-1}}s^2 < \sigma^2 < \frac{(n-1)}{\chi^2_{1-\alpha/2,n-1}}s^2$$

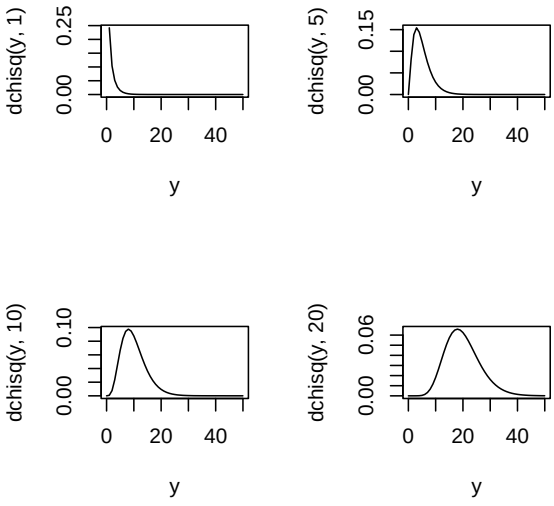

Figure 1: The density functions of $\chi^2_1, \chi^2_5, \chi^2_{10}, \chi^2_{20}$ respectively.

Review problems. Example 7.15., Excercise 7.45. Exercise 7.47. Additional problem for lecture 09.

```
> library(Devore6)
>  data(ex07.47)
> attach(ex07.47)
> ex07.47
   strength
1      11.5
2      12.1
3       9.9
4       9.3
....
> a<-(strength>10)
> a
 [1]  TRUE  TRUE FALSE FALSE...
> length(a)
[1] 48
> sum(a)
[1] 13
```