1. Suppose the fat content of a hotdog follows normal distribution. 10 measurements are given.

\[ x \leftarrow \{25.2, 21.3, 22.8, 17.0, 29.8, 21.0, 25.5, 16.0, 20.9, 19.5\} \]

We are interested in constructing interval estimate of the unknown population variance. To solve this problem, we need to know the following fact.

2. Let \( X_1, \ldots, X_n \) be a random sample from \( N(\mu, \sigma^2) \).

\[
\frac{(n-1)s^2}{\sigma^2} = \frac{1}{\sigma^2} \sum_{j=1}^{n} (X_j - \bar{X})^2 \sim \chi^2_{n-1}
\]

Quiz. What is the expectation of \( \chi^2_{n-1} \)?

\[ y \leftarrow 0:50 \]
\[ \text{par(mfrow=c(2,2))} \]
\[ \text{plot(y, dchisq(y, 1), type='l')} \]
\[ \text{plot(y, dchisq(y, 5), type='l')} \]
\[ \text{plot(y, dchisq(y, 10), type='l')} \]
\[ \text{plot(y, dchisq(y, 20), type='l')} \]

Figure 1: The density functions of \( \chi^2_1, \chi^2_5, \chi^2_{10}, \chi^2_{20} \) respectively.

3. Critical values for \( \chi^2_n \) distribution are defined as numbers that gives

\[ P(\chi^2_{1-\alpha/2,n} < \chi^2_n < \chi^2_{1+\alpha/2,n}) = 1 - \alpha \]

To find \( \chi^2_{0.975,9} \) and \( \chi^2_{0.025,9} \) that is need to construct 95\% CI for \( \sigma^2 \), we use R package:

\[ \text{qchisq(0.025, 9)} \]
\[ \text{[1]} \ 2.700389 \]
\[ \text{qchisq(0.975, 9)} \]
\[ \text{[1]} \ 19.02277 \]

4. 100(1 - \alpha)\% CI for \( \sigma^2 \):

\[
\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}
\]


\[ \text{library(Devore6)} \]
\[ \text{data(ex07.47)} \]
\[ \text{attach(ex07.47)} \]
\[ \text{ex07.47} \]
\[ \text{strength} \]
\[ 1 \ 11.5 \]
\[ 2 \ 12.1 \]
\[ 3 \ 9.9 \]
\[ 4 \ 9.3 \]
\[ \ldots \]
\[ a \leftarrow (\text{strength}>10) \]
\[ a \]
\[ \text{[1]} \ TRUE \ TRUE \ FALSE \ FALSE \ldots \]
\[ \text{length(a)} \]
\[ \text{[1]} \ 48 \]
\[ \text{sum(a)} \]
\[ \text{[1]} \ 13 \]