Stat 312: Lecture 09
Confidence intervals from normal population

Moo K. Chung
mchung@stat.wisc.edu

September 30, 2004

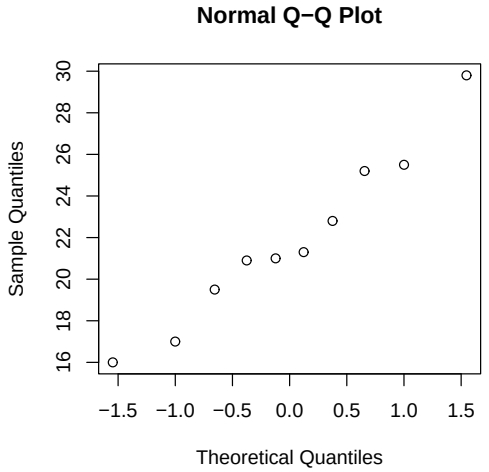

Figure 1: Normal probability plot showing normality of data.

1. Fat content of 10 randomly selected hotdogs is given by

   ```
   > x<-c(25.2,21.3,22.8,17.0,29.8,
   21.0,25.5,16.0,20.9,19.5)
   ```

   We are interested in constructing a 95% CI for the population mean fat content. Since the sample size is small, we can not apply large sample size results. Let us check the normality of data first using `qqnorm(x)`.

   So we can assume our data to follow a normal distribution but the variance is unknown.

2. If $X_i \sim N(\mu, \sigma^2)$,

   $$T \sim \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1},$$

   a t distribution with $n-1$ degrees of freedom. As $n \to \infty$, $t_n \to N(0,1)$ so for large sample size n, we can approximate t distribution with $N(0,1)$. Let us plot t distributions and compare them with $N(0,1)$.

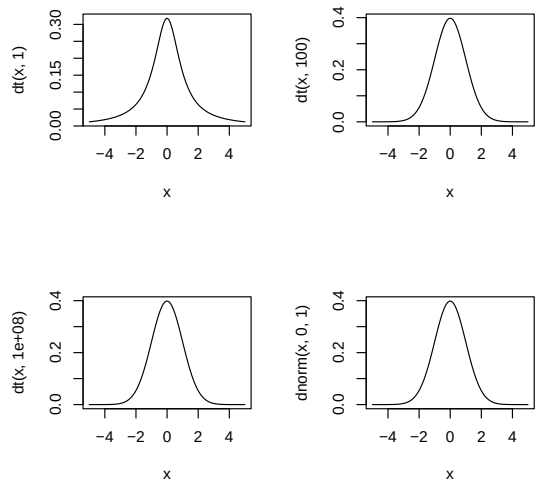

Figure 2: From left to right $t_1, t_{100}, t_{10000000}$ and $N(0,1)$.

```
> y<- -50:50/10
> par(mfrow=c(2,2))
> plot(y,dt(y,1),type='l')
> plot(y,dt(y,100),type='l')
> plot(y,dt(y,100000000),type='l')
> plot(y,dnorm(y,0,1),type='l')
```

3. Critical value $t_{\alpha,n}$ is defined as the point that gives $P(T > t_{\alpha,n}) = \alpha$. We can construct CI based on

   $$P(-t_{\alpha/2,n-1} < T < t_{\alpha/2,n-1}) = 1 - \alpha.$$

   $100(1-\alpha)\%$ CI for μ is

   $$\bar{x} \pm t_{\alpha/2} s/\sqrt{n}.$$

```
> qt(.975, df = c(1:10,20,100,1000,10000
 [1] 12.706205  4.302653  3.182449
 [4]  2.776445  2.570582  2.446912
 [7]  2.364624  2.306004  2.262157
```

```
[10]  2.228139  2.085963  1.983972
[13]  1.962339  1.960201
> qnorm(0.975)
[1] 1.959964

>
> sd(x)
[1] 4.13414
> mean(x)+qt(0.975,9)*sd(x)/sqrt(10)
[1] 24.85739
> mean(x)-qt(0.975,9)*sd(x)/sqrt(10)
[1] 18.94261

> t.test(x)
        One Sample t-test
data:  x t = 16.7517, df = 9, p-value = 4.308e-08 alternative
hypothesis: true mean is not equal to 0 95 percent confidence
interval:
 18.94261 24.85739
sample estimates: mean of x
     21.9
```

Review Problems. Exercise 7.29., 7.33.

Homework III. Due Oct 19 9:30am. Exercise 7.22.,7.26.,4.82., 4.88. part (a) only., 7.32., 7.44., 7.46.