

Stat 312: Lecture 08

Large sample confidence intervals

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1. The sample size is inversely related to the width of confidence interval. Example 7.4.

2. *Central Limit Theorem.* Let X_1, \dots, X_n be a random sample with mean μ and variance σ^2 . For sufficiently large n ,

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1).$$

3. Let X_1, \dots, X_n be a random sample with mean μ . For sufficiently large n ,

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim N(0, 1)$$

where S is the sample standard deviation. If n is sufficiently large, approximate $100(1 - \alpha)\%$ confidence interval for μ is

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}},$$

where s is the sample standard deviation.

4. *General large sample confidence interval.* Suppose $\hat{\theta}$ is an unbiased estimator of some parameter θ , Then $100(1 - \alpha)\%$ confidence interval is

$$\hat{\theta} \pm z_{\alpha/2} \sqrt{\mathbb{V}\hat{\theta}}.$$

In many applications, $\mathbb{V}\hat{\theta}$ is a function of θ which makes computation of CI complicated. In this situation, we need to estimate $\mathbb{V}\hat{\theta}$ further.

Example. Toss $n = 100$ biased coins with $P(H) = p$. Suppose you observe 38 heads. Construct 95% CI of p .

```
> X<-rbinom(100,1,0.4)
> X
[1] 1 0 0 0 0 0 0 0 1 0 1 1 0 0 0 0
[17] 0 0 1 1 0 1 0 1 1 0 1 0 1 0 1 0
[33] 1 1 0 0 0 0 1 0 1 0 0 0 0 0 1 0
```

```
[49] 0 1 0 0 0 0 1 1 0 1 0 1 1 0 0 1
[65] 0 0 1 0 0 1 0 0 0 1 0 0 1 0 0 0
[81] 1 0 1 0 0 1 1 0 0 0 1 0 0 1 1 1
[97] 0 1 1 0
> sqrt(0.38*(1-0.38)/100)*1.96
[1] 0.09513574
> 0.38+0.095
[1] 0.475
> 0.38-0.095
[1] 0.285
```

5. *One-sided confidence interval:* An $100(1 - \alpha)\%$ upper confidence bound for θ is

$$\theta < \bar{x} + z_{\alpha} \sqrt{\mathbb{V}\hat{\theta}}$$

and a lower confidence bound for μ is

$$\theta > \bar{x} - z_{\alpha} \sqrt{\mathbb{V}\hat{\theta}}.$$

Review Problems. Example 7.8, 7.10.