Stat 312: Lecture 06
Confidence Intervals

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1. A *confidence interval* (CI) is an interval used to estimate the likely size of a population parameter. A *confidence level* is a measure of the degree of reliability of the confidence interval. Most commonly used confidence levels are the 90%, 95% and 99% confidence intervals that have 0.90, 0.95 and 0.99 probabilities respectively of containing the parameter.

   Ex. For population parameter μ, 95% confidence interval $(\hat{\mu}_L, \hat{\mu}_U)$ of μ is an interval that satisfies

   $$P(\hat{\mu}_L \leq \mu \leq \hat{\mu}_U) = 0.95.$$

   We usually make the interval centered so that

   $$P(\hat{\mu}_L \leq \mu) = P(\mu \leq \hat{\mu}_U) = 0.025.$$

2. Let $X_i \sim N(\mu, \sigma^2)$ with known σ^2 and unknown μ. 95% *confidence interval* for μ is

   $$\hat{\mu}_L = \bar{x} - 1.96 \cdot \sigma/\sqrt{n},\ \hat{\mu}_U = \bar{x} + 1.96 \cdot \sigma/\sqrt{n}.$$

3. Let $X_i \sim N(\mu, \sigma^2)$ with known σ^2 and unknown μ. $100(1-\alpha)\%$ confidence interval for μ is.

   $$\hat{\mu}_L = \bar{x} - z_{\alpha/2} \cdot \sigma/\sqrt{n},\ \hat{\mu}_U = \bar{x} + z_{\alpha/2} \cdot \sigma/\sqrt{n},$$

   where quantile $z_{\alpha/2}$ is given by

   $$P\big(Z > z_{\alpha/2}\big) = \frac{\alpha}{2}.$$

4. A 95% confidence interval can be interpreted probabilistically as an interval that can contain true unknown parameter 95% of time in the long run. The following simulation demonstrates this.
   Let $X_i \sim N(\mu, 14^2)$. Suppose $\mu = 42$ but assume we do not know this fact. Let $n = 10$. This is the binge drinking example of Lecture 1.

```
> b=1.96*14/sqrt(10)
> b
[1] 8.67729

> a1<-rnorm(10,42,14)
> a1
  [1] 72.97298 39.29226 48.76871 52.
> c(mean(a1)-b,mean(a1)+b)
[1] 38.63614 55.99072

>  a2<-rnorm(10,42,14)
> c(mean(a2)-b,mean(a2)+b)
[1] 30.18341 47.53799

> c(mean(a3)-b,mean(a3)+b)
[1] 32.18435 49.53893

>  a4<-rnorm(10,42,14)
> c(mean(a4)-b,mean(a4)+b)
[1] 40.65048 58.00506

... If you this many many times ....

>  a17<-rnorm(10,42,14)
> c(mean(a17)-b,mean(a17)+b)
[1] 45.15059 62.50517
```

Review problems. Example 7.2.,7.3.