Stat 312: Lecture 02
Point Estimation

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September 2, 2004

1. If X and Y are independent, $\mathbb{E}(X+Y) = \mathbb{E}X + \mathbb{E}Y$ and $\mathbb{V}(X+Y) = \mathbb{V}X + \mathbb{V}Y$. See pp. 244.

2. If a random sample $X_i \sim N(\mu, \sigma^2)$, $\bar{X} \sim N(\mu, \sigma^2/n)$.
   Proof.
   $$\begin{aligned} \mathbb{V}\bar{X} &= \mathbb{E}\Big(\frac{\sum X_i}{n}\Big)^2 - \Big[\mathbb{E}\Big(\frac{\sum X_i}{n}\Big)\Big]^2 \\ &= \frac{1}{n^2}\mathbb{E}(\sum X_i)^2 - \mu^2. \end{aligned}$$
   Now note that
   $$\begin{aligned} \mathbb{E}(\sum X_i)^2 &= \mathbb{V}(\sum X_i) + (\mathbb{E}\sum X_i)^2 \\ &= \sum \mathbb{V}X_i + (\sum \mathbb{E}X_i)^2 \\ &= n\sigma^2 + (n\mu)^2 \end{aligned}$$
   Hence $\mathbb{V}\bar{X} = \frac{\sigma^2}{n}$.

3. A *point estimate* of population parameter θ is a single number that reasonably approximate θ. A *point estimator* of θ is a statistic (rule or formula) for getting the point estimate given sample data.

4. Let X_i be a Bernoulli random variable with $P(X_i = 1) = p$. Then $\sum_{i=1}^n X_i$ is a Binomial random variable with n and p parameters. Note that a Binomial random variable with $n = 1$ is a Bernoulli random variable. The following example simulates 1000 coin tosses.

```
> a<-rbinom(1000,1,0.5)
> a
   [1] 0 0 1 0 0 1 0 1 1 0 0 ...
> sum(a)/1000
[1] 0.517
> b<-rbinom(1000000,1,0.5)
> sum(b)/1000000
[1] 0.500545
```

5. We can let
   $$\hat{\theta} = \theta + \text{error of estimation}.$$
   Taking expectation on both sides, we define the *bias* of estimator $\hat{\theta}$ to be
   $$Bias(\hat{\theta}) = \mathbb{E}(\hat{\theta}) - \theta.$$
   It measures the biasness of estimator $\hat{\theta}$ in average sense.

Reviw Problems. example 6.1.,6.3.
Assignment I. Due Sept 16 9:30am. Exercise. 6.2. 6.8. 6.10. 6.16. 6.20. 6.22.