Surface-based Morphometry (SBM)

• Surface-specific morphometric technique.

• More sensitive to surface-specific changes.

• Example: substantial cortical changes were found in AD and other dementias and it is likely that different clinical populations will exhibit different cortical shape variability.
Motivation for SBM

Compared to other 3D volumetric techniques, SBM can quantify cortical variations better.

Ventricle enlargement

Age 14

Age 19
Comparison of cortical surface

Superimposition
MNI cortical image processing pipeline

- Native MRI
- Resampling
- Classification
- Segmentation
- Partial Volume Estimation
- white-surface deformation
- expand to grey surface
- cortical thickness
- diffusion smoothing
- Non-uniformity correction
- Linear Registration
- non-linear registration
- Gaussian Blur
- Asymmetry maps
- Gaussian Blur
Image intensity Nonuniformity correction N3 algorithm

Original data

Corrected

Source: Jason Lerch, MNI
Tissue segmentation: neural network classifier

Original image

3 disjoint classes
Assigned values: 1, 2 and 3
Deformable surface algorithm McDonalds et al. (2001) NeuroImage

Multiscale triangle subdivision at each iteration increases the complexity of cortical folding.
Two available cortical thickness analysis software

FreeSurfer: Bruce Fischl
http://surfer.nmr.mgh.harvard.edu

BrainVista: J.F. Mangin
http://brainvisa.info
Final result gives two cortical surfaces

Yellow: outer cortical surface
Blue: inner cortical surface
Final surface extraction result

Inner surface

Outer surface
Polygonal mesh data structure
Basis of most surface rendering tools for 3D computer games:
as 3D Max Studio, Maya
Data structure for polygonal mesh

Coordinates for subject 1

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Coordinates for subject 2

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Corresponding vertices have approximate anatomical homology.
Surface Registration

In order to compare cortical thickness across subjects, it is necessary to find a mapping between homologous anatomical regions.

Sulcal pattern alignment = finger print of brain
Compute cortical curvature and map curvature to unit sphere.

Unit sphere gives a natural coordinate system (spherical coordinates).
Sulcal pattern matching by minimizing objective function = curvature difference - smoothness of deformation.
Average Template construction
Averaged corresponding coordinates of individual surface

Top: Old technique Chung et al. (2003)
Bottom: New technique Chung et al. (2005)
Demonstrating the alignment of sulci on average template

Principle curvature maps projected on the average template
Validation of surface registration based on 149 subjects

Central and temporal sulci

Traces of sulci provided by Cathia et al. (2003)
Right central sulcus matching probability

3D volume registration

2D surface registration
Alternate approach: spherical harmonic (SPHARM) correspondence

- Surface registration is given implicitly by matching the coefficients of basis functions of deformation.

- Since the deformation is given in terms of smooth basis function, we do not need to worry about increasing the smoothness of deformation.

- This is shown to be optimal in the least squares fashion.

- SPHARM will be discussed in lecture 6
Measuring cortical thickness

thickness of gray matter is used as an anatomical index.

orthogonal projection from A to B  orthogonal projection from B to C
Cortical thickness measurements

\[ Y(p) = \theta(p) + \epsilon(p), \quad p \in \partial \Omega \]

where $\theta$ is the mean thickness and $\epsilon$ is a zero mean Gaussian random field.
Noisy thickness measures from triangle mesh

Flattened map:
Spherical projection
(visualization only)
Flat map of cortical thickness

Original thickness

Heat kernel smoothing
Why do we smooth images before statistical analysis?
1. To increase the signal-to-noise ratio (SNR).
2. Random field based multiple comparison correction requires very smooth Gaussian random field assumption.

Why Gaussian kernel smoothing?
1. It is a standard technique in imaging.
2. Computationally fast.
3. Computationally easy to implement.

How to perform “kernel smoothing” on anatomical boundary?
Difficulty of performing smoothing along boundary

Due to curved geometry, the shortest distance between two points is not a straight line. So we may incorrectly assign less weights to the closer measurements.

Improper kernel weighting

Proper kernel weighting
Smoothing on Anatomical Boundary

Gaussian kernel smoothing

\[ F(x, t) = \frac{1}{(4\pi t)^{n/2}} \int_{\mathbb{R}^n} e^{-(x-y)^2/4t} f(y) \, dy \]

Isotropic diffusion

\[ \frac{\partial F}{\partial t} = \Delta F = \frac{\partial^2 F}{\partial^2 x_1} + \cdots + \frac{\partial^2 F}{\partial^2 x_n}, \quad F(x, 0) = f(x) \]
Heat kernel smoothing

Gaussian kernel smoothing is a special case of heat kernel smoothing. It is the solution of the isotropic diffusion equation.

$K_\sigma \ast Y(p) = \int_{\partial \Omega} K_\sigma(p, q) Y(q) \, d\mu(q)$

Estimate heat kernel $K$ and simply perform integral convolution on surface.

Mathematical detail will be discussed later.
Iterated heat kernel identity

**Theorem 4** Heat kernel smoothing with large bandwidth can be decomposed into multiple kernel smoothing with smaller bandwidth via

\[ K^{(k)}_\sigma \ast f = K_\sigma \ast \cdots \ast K_\sigma \ast f \overset{k \text{ times}}{=} K_{\sqrt{k} \sigma} \ast f. \]

**Example.**
Single Gaussian kernel smoothing with the bandwidth of **10 mm FWHM**.

= **100** repeated applications of Gaussian kernel smoothing with the bandwidth of **1 mm FWHM**.
1st order approximation of heat kernel

\[
\tilde{W}_\sigma(p, q_i) = \frac{\exp \left[ - \frac{d^2(p, q_i)}{2\sigma^2} \right]}{\sum_{j=0}^{m} \exp \left[ - \frac{d^2(p, q_j)}{2\sigma^2} \right]}
\]

Heat kernel smoothing is performed iteratively with smaller FWHM.

**Algorithm 1**

For \( i = 1 \) to \( n \) do

Find a set of neighboring vertices \( N(q_i) \) of \( q_i \).

Compute the weighted average and store \( Z(q_i) \leftarrow W_\sigma \ast Y(q_i) \).

End.

Update \( Y \leftarrow Z \).

Repeat this procedures \( k \)-times.
Heat kernel smoothing on cortical thickness
corrected $p$-value map for $t$-test in autism

**Decrease:** left superior temporal sulcus, left occipital-temporal gyrus, right orbital prefrontal

**Increase:** left superior temporal gyrus, left middle temporal gyrus, left and right postcentral sulci
corrected $p$-value map for $F$ test accounting for age effect

**Decrease:**
- left superior temporal sulcus
- left occipital-temporal gyrus
- right orbital prefrontal
Cortical thickness change $t$ map between age 12 and 16.
Other surface measures

- cortical thickness, curvatures, surface area, tissue density.

- fractal dimension = measure of complexity of anatomical shape.

- surfaces of most anatomical boundaries: corpus callosum, hippocampus, amygdala, liver, mandible.
Curvature estimation via Surface Parameterization

Global: tensor splines, SPHARM
Local: quadratic surface fitting

\[ X(u^1, u^2) = \begin{pmatrix} x_1(u^1, u^2) \\ x_2(u^1, u^2) \\ x_3(u^1, u^2) \end{pmatrix} \]

Find the best fitting tangent plane via PCA

\[ s(u^1, u^2) = \beta_1 u_1 + \beta_2 u_2 + \beta_3 u_1^2 + 2\beta_4 u_1 u_2 + \beta_5 u_2^2 + \cdots \]
Polynomial Regression on irregular triangular mesh

\[ Y = X\beta \]

\[
\begin{pmatrix}
  u_1^3 \\
  u_2^3 \\
  \vdots \\
  u_m^3
\end{pmatrix} =
\begin{pmatrix}
  u_1^1 & u_1^2 & (u_1^1)^2 & u_1^1 u_1^2 & (u_1^2)^2 \\
  u_2^1 & u_2^2 & (u_2^1)^2 & u_2^1 u_2^2 & (u_2^2)^2 \\
  \vdots & \vdots & \vdots & \vdots & \vdots \\
  u_m^1 & u_m^2 & (u_m^1)^2 & u_m^1 u_m^2 & (u_m^2)^2
\end{pmatrix}
\begin{pmatrix}
  \beta_1 \\
  \beta_2 \\
  \vdots \\
  \beta_m
\end{pmatrix}
\]

\[ \bar{\beta} = (X'X)^{-1}X'Y \]
Riemannian Metric Tensors

The first fundamental form

\[ g_{ij} = \langle \frac{\partial X}{\partial u^i}, \frac{\partial X}{\partial u^j} \rangle \]

The second fundamental form

\[ l_{ij} = \langle n, \frac{\partial^2 X}{\partial u^i \partial u^j} \rangle \]

Mean curvature

\[ K_M = \text{tr}(g^{-1}l)/2 \]

Gaussian curvature

\[ K_G = \det(g^{-1}l) = \det(l)/\det(g) \]

Laplace-Beltrami operator

\[ \Delta_X F = \frac{1}{|g|^{1/2}} \sum_{i,j=1}^{2} \frac{\partial}{\partial u^i} \left( |g|^{1/2} g^{ij} \frac{\partial F}{\partial u^j} \right) \]
Why quadratic surface?

\[ s(u^1, u^2) = \beta_1 u_1 + \beta_2 u_2 + \beta_3 u_1^2 + 2\beta_4 u_1 u_2 + \beta_5 u_2^2 + \cdots \]

\[
g = \begin{pmatrix}
1 + \beta_1^2 & \beta_1 \beta_2 \\
\beta_1 \beta_2 & 1 + \beta_2^2
\end{pmatrix}
\]

\[
l = \begin{pmatrix}
\beta_3 \\
\beta_4 \\
\beta_5
\end{pmatrix}
\]

\[
K_M = \frac{\text{tr}(g^{-1}l)}{2} = \frac{\beta_3(1 + \beta_2^2) + \beta_5(1 + \beta_1^2) - 2\beta_1 \beta_2 \beta_4}{2 + 4(\beta_1^2 + \beta_2^2)}
\]
Bending energy or thin-plate spline energy can be used to measure the curvature of the surface. Between ages 12 and 16, it increases both locally and globally.
Curvature change $t$ map between age 12 and 16
Surface area expansion/shrinking

Local surface area element:

$$\sqrt{|g|} = \sqrt{1 + \beta_1^2 + \beta_2^2}$$
Local area expansion with respect to a template (it ranges between 0 and 1.3)
Surface area change $t$ map

dilatation rate between age 12 and 16

$\min = -57\% \quad \text{mean} = -0.02\% \quad \max = 65\%$
Lecture 6 Topics

Spherical harmonic (SPHARM) representation