

STATISTICS 311**Discussion #1****September 8, 2006****I. Calculus Review**

1. The Fundamental Theorem of Calculus

If $f(x)$ is the derivative of $F(x)$, that is $f = F'$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

2. The Short Table of Integrals

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
<i>Constant</i> (a)	$ax + C$	a^x	$\frac{a^x}{\ln a} + C$
ax^b	$\frac{ax^{b+1}}{b+1} + C$	e^x	$e^x + C$
$\sin x$	$-\cos x + C$	$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x + C$
$\cos x$	$\sin x + C$	$\frac{1}{1+x^2}$	$\arctan x + C$
$\tan x$	$-\ln \cos x + C$	$\frac{1}{x^2-1}$	$\frac{1}{2} \ln \left \frac{x-1}{x+1} \right + C$
$\frac{1}{\cos^2 x}$	$\tan x + C$	$\frac{1}{x}$	$\ln x + C$
$\frac{1}{\sin^2 x}$	$-\cot x + C$	$\ln x$	$x \ln x - x + c$

3. Integration by Parts

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

4. Integration by Substitution

If an integral is of form $\int_a^b f(m(x))m'(x)dx$, then let

$$w = m(x)$$

$$dw = m'(x)dx$$

the transformed integral will be

$$\int_{m(a)}^{m(b)} f(w)dw$$

5. Properties

(a) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, here a, b, c are known constants, and usually $a \leq c \leq b$

(b) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

6. Theorem of Maximum and Minimum Value

Suppose that the function $f(x)$ is a continuous function defined on the closed interval $[a, b]$, and is differentiable on the open interval (a, b) . Suppose that x_1, x_2, \dots, x_n are all the stationary points of $f(x)$ within (a, b) . Then among the values:

$$f(a), f(x_1), \dots, f(x_n), f(b)$$

the largest one is the Maximum Value of $f(x)$, and the smallest one is the Minimum Value of $f(x)$.

Note that the point x on which $f'(x) = 0$ is the stationary point of $f(x)$.

II. Practice Problems

1. Calculate $\int_0^{\infty} xe^{-x}dx$.

2. Calculate $\int_0^{\infty} x \frac{3}{(1+x)^4} dx$

3. Let $f(x, y) = e^{x+2y}$. Calculate $\int \int_R f(x, y) dx dy$ where R is the region in the plane in which (x, y) satisfies $y \leq x < 0$.

4. Let

$$f(x, y) = \begin{cases} \frac{x+2y}{256}, & 2 \leq x \leq 4, 2 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy$.

5. Find the value of $x \geq 0$ which maximizes $x^2 e^{-3x}$.