STATISTICS 311 Discussion #7 October 23, 2006

I. Review: Joint Distribution

1. Discrete Case: X, Y are discrete random variables with $R(X)$ and $R(Y)$

   (a) Joint probability mass function(p.m.f.) is

   $$P(X = x, Y = y) = p_{XY}(x, y) \quad x \in R(X),\ y \in R(Y)$$

   where $P(X = x, Y = y) \geq 0$, and $\sum_{x \in R(X)} \sum_{y \in R(Y)} P(X = x, Y = y) = 1$.

   (b) If X and Y have joint p.m.f. $P(X = x, Y = y)$, then

   $$E[h(X, Y)] = \sum_{x \in R(X)} \sum_{y \in R(Y)} h(x, y) P(X = x, Y = y)$$

   (c) If X and Y are Independent, then

   $$P(X = x, Y = y) = P(X = x)P(Y = y)$$

2. Continuous Case

   (a) Joint Density Function

   - $f_{XY}(x, y) \geq 0$
   - $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x, y) dx dy = 1$

   (b) Joint Distribution Function

   $$F_{XY}(x, y) = P(X \leq x, Y \leq y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{XY}(x, y) dx dy$$

   (c) Marginal Density Function

   $$f_x(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad \text{and} \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx \ .$$

   (d) If X and Y have joint density function $f_{XY}(x, y)$, then

   $$E[h(X, Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) f_{XY}(x, y) dx dy$$

   (e) If X and Y are Independent, then

   $$F_{XY}(x, y) = F_X(x)F_Y(y) \quad \text{or} \quad f_{XY}(x, y) = f_X(x)f_Y(y)$$

II. Examples

1. Problem 8 on page 290.

2. Problem 9 on page 290.

3. Let X and Y have joint pdf

$$f_{XY}(x,y) = \begin{cases} \frac{1}{2}xy, & 0 \le y \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Compute the density function of X.

   (b) Find $P(X < 2Y)$.

   (c) Find $P(Y > 1|X > 1)$.

   (d) Find the distribution of $Z = X/Y$.

if possible:

Problem 14 on page 291

Problem 27 on page 293

Problem 28 on page 293