STATISTICS 311 Discussion #5 October 10, 2006

I. Review

1. Bernoulli Trials

   - Each trial has two possible outcomes (Success: S, Failure: F).
   - Trials are independent.
   - Same probability of success at each trial.

2. Binomial Distribution with Parameters n and p

   - Definition: Binomial r.v. is the total number of successes in n Bernoulli trails each having $P(S) = p$. The probability distribution of the Binomial r.v. is the Binomial distribution.
   - Notation: $X \sim \text{Bin}(n, p)$.
   - Calculations of Binomial distribution:

$$P(X = k) = \binom{n}{k} p^k q^{n-k} \qquad k = 0, 1, \cdots, n$$

   - $E(X) = np, \quad Var(X) = npq = np(1-p)$

3. Continuous Random Variable

   - Definition: X is a continuous random variable, if there is a nonnegative f, defined for all real $X \in (-\infty, \infty)$ having the property that for any set B of real number,

$$P(X \in B) = \int_B f(x)\mathrm{d}x$$

     This $f(x)$ is called the probability density function (p.d.f) of X.

     (a) $f(x) \geq 0$,

     (b) $\int_{-\infty}^{\infty} f(x)\mathrm{d}x = 1$

   - Cumulative Distribution Function

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)du$$

$$P(a \leq X \leq b) = F(b) - F(a)$$

   - $f(x) = F'(x)$

- P(X=a)=0
- Expected Value:

$$E(X) = \int_x xf(x)\mathrm{d}x$$

$$E[g(X)] = \int_x g(x)f(x)\mathrm{d}x$$

- Variance:

$$Var(X) = E(X-\mu)^2 = \int_x (x-\mu)^2 f(x)\mathrm{d}x$$

$$Var(X) = E(X^2) - \mu^2 = \int_x x^2 f(x)\mathrm{d}x - \mu^2$$

II. Examples

1. Problem 26 on Page 174.
2. Problem 38 on page 176.
3. Problem 41 on Page 176.
4. Problem 42 on Page 176.
5. Problem 44 on page 176.