

**STATISTICS 311****Discussion #2****September 18, 2006****I. Probability Review**

## 1. Fundamental Concepts

- (a) Sample Space: the set of all possible outcomes, denoted by  $S$ .
- (b) Events: the event corresponding to a statement about the outcome is the collection of elements for which the statement is true.
- (c) Probability Measure: A function defined on the collection of event with the following properties:
  - $0 \leq P(A) \leq 1$ ,  $A$  is the event of the sample space  $S$ .
  - $P(S) = 1$
  - If  $A$  and  $B$  are disjoint, then  $P(A \cup B) = P(A) + P(B)$ . If  $A_1 \cdots A_m$  are disjoint, then  $P(\bigcup_{i=1}^m A_i) = \sum_{i=1}^m P(A_i)$ .

## 2. Some Properties

- (a)  $P(A^c) = 1 - P(A)$
- (b)  $P(\emptyset) = 0$
- (c) If  $A \subset B$ , then  $P(A) \leq P(B)$
- (d) Addition Law:  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ . Especially, if  $A \cap B = \emptyset$ , then  $P(A \cup B) = P(A) + P(B)$ .
- (e)  $(A \cap B)^c = A^c \cup B^c$ , or  $(A \cup B)^c = A^c \cap B^c$ .
- (f)  $A \cup B = (A \cap B^c) \cup B$ .
- (g)  $A = (A \cap B) \cup (A \cap B^c)$ .

**II. Counting Methods for Computing Probabilities**

- 1. Uniform probability model: If  $S = \{w_1, w_2, \dots, w_n\}$ ,  $P(w_1) = P(w_2) = \dots = P(w_n) = \frac{1}{n}$ , then the probability of  $A$  is  $P(A) = \frac{\# \text{ of } A}{\# \text{ of } \Omega}$ .

## 2. Multiplication Principle:

If one experiment has  $m$  outcomes and another experiment has  $n$  outcomes, then there are  $mn$  possible outcomes for the two experiments.

### 3. Permutation

Permutation is an ordered arrangement of outcomes. Suppose we have  $n$  objects and choose  $r$  objects and list them in order,

with replacement:  $n^r$

without replacement:

$$\frac{n!}{(n-r)!} = n(n-1)(n-2)\dots(n-r+1)$$

### 4. Combination

If  $r$  objects are taken from  $n$  objects without replacement and disregarding the order,

$$\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n \times (n-1) \times \dots \times (n-r+1)}{r \times (r-1) \times \dots \times 1}$$

Properties:

$$(1). \binom{n}{r} = \binom{n}{n-r}$$

$$(2). \binom{n}{n} = \binom{n}{0} = 1$$

$$(3). \binom{n}{1} = \binom{n}{n-1} = n$$

## III. Examples

1. Problem 6 on Page 54.
2. Problem 17 on Page 55.
3. A pair of fair dice are rolled. What is the probability that the first die lands on a higher value than does the second?
4. Problem 37 on Page 57.
5. Problem 39 on Page 57.
6. Problem 42 on Page 57.