STAT572: EXPERIMENTAL DESIGNS

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Experiment: The Tea Tasting Lady

The lady of the Rothamsted Experiment Station staff claimed that she could discriminate between a cup of tea made with milk and one with tea added first. Fishers experiment design consisted of making 8 cups of tea with 4 made in one way and 4 in the other. The lady was told of this structure. The 8 cups are presented to the lady in random order and she has to partition the 8 cups into two sets of 4. The interpretation will be made on the basis that there are 70 partitions. So, if the assignment was random, the probability of the lady obtaining the correct partition is 1/70, if she cannot discriminate. Because 1/70 is a small probability, it is rational to conclude that if she obtains the correct partition she has given evidence in favor of her claim. (R. A. Fisher, The Design of Experiments, 1935.)

Definitions

- Experiment : an operation or procedure carried out under controlled conditions in order to discover an unknown effect or law, to test or establish a hypothesis, or to illustrate a known law (Merriam-Webster).
- Factor : variable whose influence upon a response variable is being studied in the experiment.
- Factor Level : numerical values or settings for a factor.
- Trial (or run) : application of a treatment to an experimental unit.
- Treatment or level combination : set of values for all factors in a trial.
- Experimental unit : object to which a treatment is applied.
- Design : a set of carefully chosen factor values used for conducting the experiment.

A Systematic Approach to Experimentation

- State the objective of the study.
- Choose the response variable: should correspond to the purpose of the study.
- Nominal-the-best, larger-the-better or smaller-the-better.
- Choose factors and levels.
- Choose experimental design (i.e., plan).
- Perform the experiment (use a planning matrix to determine the set of treatments and the order to be run).
- Analyze data (design should be selected to meet objective so that the analysis is efficient and easy).
- Draw conclusions.

Fundamental Principles:Replication

- Each treatment is applied to units that are representative of the population (example : measurements of 3 units vs. 3 repeated measurements of 1 unit).
- Replication vs Repetition (i.e., repeated measurements).
- Enable the estimation of experimental error. Use sample standard deviation.

Fundamental Principles:Randomization

- Use of a chance mechanism (e.g., random number generators) to assign treatments to units or to run order. It has the following advantages.
- Protect against latent variables or lurking variables.
- Reduce influence of subjective bias in treatment assignments (e.g., clinical trials).

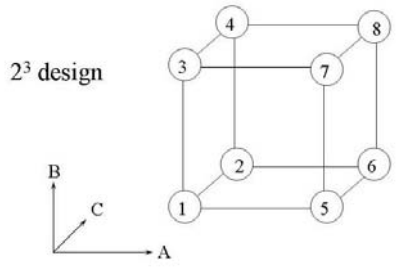
Fundamental Principles:Blocking

- A block refers to a collection of homogeneous units. Effective blocking : larger between-block variations than within-block variations. (Examples: hours, batches, lots, street blocks, pairs of twins.)
- Run and compare treatments within the same blocks. (Use randomization within blocks.) It can eliminate block-block variation and reduce variability of treatment effects estimates.
- **Block what you can and randomize what you cannot**.

Full Factorial Design

- **Goal:** Explore the possible combinations of settings effectively. Successfully investigate the effects of two or more factors simultaneously.
- 2^k **Design**
  1. k factors: Total k factors are involved in the experiment.
  2. 2 levels: We assume two levels of treatment for each factor.
  3. 2^k Design: So we will observe at least $2 \times 2 \times \cdots \times 2 = 2^k$ observations.

2^3 Design



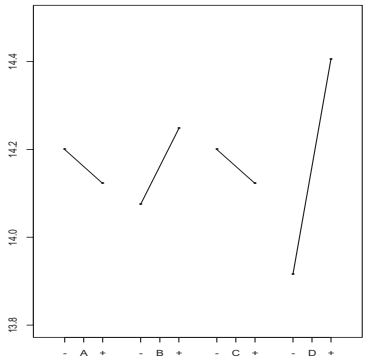
Epitaxial Layer Growth Experiment

- An AT&T experiment based on 2^4 design; four factors each at two levels. There are 16 ($=2^4$) level combinations.

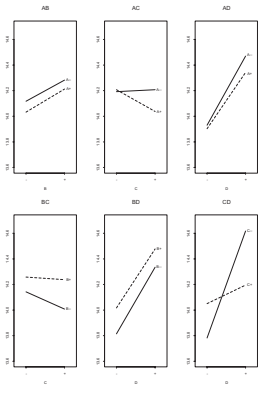
Table: Factors and Levels, Layer Growth Experiment

Factor		Level	
		−	+
A.	susceptor-rotation method	continuous	oscillating
B.	code of wafers	668G4	678D4
C.	deposition temperature(°C)	1210	1220
D.	deposition time	short	long

Main Effects



Interaction Effects



One-Factor-At-A-Time(OFAT)

- **OFAT approach**
  1. Identify the most important factor
  2. investigate this factor by itself, ignoring the others
  3. make recommendation on changes(or no change)
  4. move onto the next important factors.

OFAT and Full factorial design

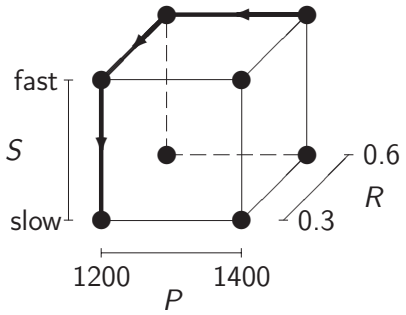


Figure: The Path of a One-Factor-At-A-Time Plan

Completely Randomized Design

A pharmaceutical company conducts an experiment to compare 5 drugs. 30 animals are available for the trial. Each drug is injected into 6 randomly selected animals. All the animals are very similar. After an appropriate period of time 2 blood samples are taken from each animal and duplicate analyses are made for each blood sample. The reading from each analysis represents the observation to be used for the statistical analysis of this experiment.

Randomized Block Designs

A study is contemplated to examine the effectiveness of three methods to memorize German vocabulary at the high school level (Kirk, 1982). It seems reasonable to take student ability, as measured by IQ, and gender into account. Thus, one approach would be to set up, say, five IQ classes, with three students for each class and gender, leading to ten blocks. In each block the three methods under investigation will then be assigned randomly to the three students.

Latin Square Designs

A manufacturer wants to investigate and compare different production processes for ceramic cookware. Extraneous sources of systematic variability are identified as (i) different batches of raw material and (ii) different ovens used for baking the product. The batches of material are obtained from different sources and possibly at different times. The ovens available in the manufacturing plant are of different makes and different ages. We have 4 different processes, 4 different batches, and 4 different ovens.

Latin Square Designs

	Oven1	Oven2	Oven3	Oven4
Batch1	D	B	C	A
Batch2	C	A	B	D
Batch3	B	D	A	C
Batch4	A	C	D	B