1 Point Estimation

1.1 Exercise 6.1.15

(Notice this is a homework problem) Let $X_1, X_2, \ldots, X_n$ represent a random sample from a Rayleigh distribution with pdf

$$f(x; \theta) = \frac{x}{\theta} e^{-x^2/(2\theta)} \text{ for } x > 0$$

(a) It can be shown that $E(X^2) = 2\theta$. Use this fact to construct an unbiased estimator of $\theta$ based on $\sum_{i=1}^{n} X_i^2$.

(b) Estimate $\theta$ from the following $n = 10$ observations on vibratory stress of a turbine blade under specified conditions:

$$
\begin{array}{ccccccc}
16.88 & 10.23 & 4.59 & 6.66 & 13.68 \\
14.23 & 19.87 & 9.40 & 6.51 & 10.95 \\
\end{array}
$$

1.2 Exercise 6.2.28

Let $X_1, X_2, \ldots, X_n$ represent a random sample from Rayleigh distribution with density function given in Exercise 15. Determine

(a) The maximum likelihood estimator of $\theta$, and then calculate the estimate for the vibratory stress data given in that exercise. Is this estimator the same as the unbiased estimator suggested in Exercise 15?

(b) The MLE of the median of the vibratory stress distribution. *Hint:* first express the median in terms of $\theta$.

1.3 Exercise 6.1.16

Suppose the true average growth $\mu$ of one type of plant during a 1-year period is identical to that of a second type, but the variance of growth for the first type is $\sigma^2$, whereas for the second type the variance is $4\sigma^2$. Let $X_1, X_2, \ldots, X_m$ be $m$ independent growth observations on the first type (so $E(X_i) = \mu$ and $\text{Var}(X_i) = \sigma^2$) and let $Y_1, Y_2, \ldots, Y_n$ be $n$ independent growth observations of the second type (so $E(Y_i) = \mu$ and $\text{Var}(Y_i) = 4\sigma^2$).

(a) Show that for any $\delta \in [0, 1]$, the estimator $\hat{\mu} = \delta \bar{X} + (1 - \delta) \bar{Y}$ is unbiased for $\mu$.

(b) For fixed $m$ and $n$, compute $\text{Var}(\hat{\mu})$ and then find the value of $\delta$ that minimizes it.

1.4 Exercise 6.2.27

(Notice this is a homework problem) Let $X_1, X_2, \ldots, X_n$ be a random sample from a gamma distribution with parameters $\alpha$ and $\beta$.

(a) Derive the equations whose solutions yield the maximum likelihood estimators of $\alpha$ and $\beta$. Do you think they can be solved explicitly?

(b) Show that the MLE of $\mu = \alpha \beta$ is $\hat{\mu} = \bar{X}$.

*Extra credit:* Find the method of moments estimators of $\alpha$ and $\beta.$