Discussion Session # 6

1 The Normal Distribution

If \( Z \sim N(0,1) \), then there’s no known analytical form for \( F(z) = \mathbb{P}(Z \leq z) \). There are tables for numerical values of it (Pages 521 and 522 of the book).

Remark: if \( Z \sim N(0,1) \) and \( z \) is such that \( \mathbb{P}(Z \leq z) = \alpha \) (the so-called \( \alpha \)-quantile of \( Z \)), then for \( X \sim N(\mu, \sigma^2) \), we can find the \( \alpha \)-quantile of \( X \) by taking \( x = \sigma z + \mu \). Also, keep in mind that

\[
\frac{X - \mu}{\sigma} = Z \sim N(0,1),
\]

\[
\sigma Z + \mu = X \sim N(\mu, \sigma^2).
\]

2 Exercise 4.3.7

The lifetime of a light bulb in a certain application is normally distributed with mean \( \mu = 1400 \) hours and standard deviation \( \sigma = 200 \) hours.

(a) What is the probability that a light bulb will last more than 1800 hours?

(b) Find the 10\(^{th}\) percentile of the lifetimes.

(c) A particular battery lasts 1645 hours. What percentile is its lifetime on?

(d) What is the probability that the lifetime of a battery is between 1350 and 1550 hours?

3 The LogNormal Distribution

If \( X \sim N(\mu, \sigma^2) \), then \( Y = e^X \) has LogNormal Distribution with parameters \( \mu \) and \( \sigma^2 \). The expected value of \( Y \) is given by

\[
\mathbb{E}(Y) = e^{\mu + \sigma^2}/2
\]

The variance of \( Y \) is given by

\[
\text{Var}(Y) = e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2}
\]

4 Exercise 4.4.4

The article “Stochastic Estimates of Exposure and Cancer Risk from Carbon Tetrachloride Released to the Air from the Rocky Flats Plant” (Rood \textit{et al.}, 2001) models the increase in the risk of cancer due to exposure to carbon tetrachloride as lognormal with \( \mu = -15.65 \) and \( \sigma = 0.79 \).

(a) Find the mean risk.

(b) Find the median risk.

(c) Find the standard deviation of the risk.

(e) Find the 95\(^{th}\) percentile.

(f) (extra) What is the probability of having a risk of cancer increase greater than 3?
5 Central Limit Theorem

Let $X_1, \ldots, X_n$ be a large random sample of a population with mean $\mu$ and variance $\sigma^2$. Then

$$\bar{X} = \frac{X_1 + \ldots + X_n}{n} \approx N \left( \mu, \frac{\sigma^2}{n} \right)$$

and

$$\sum_{i=1}^{n} X_i = X_1 + \ldots + X_n \approx N \left( n\mu, n\sigma^2 \right)$$

6 Exercise 4.8.11

A new process has been designed to make the ceramic tiles. The goal is to have no more than 5% of the tiles be nonconforming due to surface defects. A random sample of 1000 tiles is inspected. Let $X$ be the number of nonconforming tiles in the sample.

(a) If 5% of the tiles produced are nonconforming, what is $P(X \geq 75)$?

(b) Based on the answer for part (a), if 5% of the tiles are nonconforming, is 75 tiles out of 1000 an unusually large number?