Discussion Session # 4

1 Conditional Probability

Let $A$ and $B$ be two events. Then if $P(B) \neq 0$, the conditional Probability of $A$ given $B$ is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

2 Exercise 3.2.2

A drag races has two parachutes, a main and a backup, that are designed to bring the vehicle to a stop after the end of the run. Suppose that the main chute deploys with probability 0.99 and that if the main fails to deploy, the backup deploys with probability 0.98.

(a) What is the probability that one of the two parachutes deploys?

(b) What is the probability that the backup parachute deploys?

(c) Additional question: What is the probability that in 100 deploys the parachute never fails?

3 Exercise 3.2.10

If $A$ and $B$ are independent events, prove that the following pairs of events are independent: $A^c$ and $B$.

4 Random Variables

For random variables taking values in the integers $\mathbb{Z}$, the probability mass function $P(X = x)$ is a non-negative function which adds to 1. The expected value of $X$ is given by

$$E(X) = \sum_i x_i P(X = x_i).$$

For random variables taking values in the real line $\mathbb{R}$, the probability density function $f(x)$ is a non-negative function which integrates to 1. The expected value of $X$ is given by

$$E(X) = \int x f(x) dx.$$

5 Exercise 3.3.8

Elongation (in %) of steel plates treated with aluminum are random with probability density function proportional to

$$f(x) = \begin{cases} cx, & 20 < x < 30, \\ 0, & \text{otherwise} \end{cases}$$

(a) For which constant $c$ is $f(x)$ a probability density function?

(b) What proportion of steel plates have elongations greater than 25%?

(c) Find the mean elongation.
6  Expectation and Standard Deviation of functions of Random Variables

If $X_1, X_2$ are independent random variables with means $\mu_1, \mu_2$ and variances $\sigma^2_1, \sigma^2_2$, respectively, then

$$E(aX_1 + bX_2 + c) = aE(X_1) + bE(X_2) + c = a\mu_1 + b\mu_2 + c,$$

$$\text{Var}(aX_1 + bX_2 + c) = a^2\text{Var}(X_1) + b^2\text{Var}(X_2) = a^2\sigma^2_1 + b^2\sigma^2_2.$$

If $f$ is a non-linear function differentiable at $\mu_1$, then these results are approximately true:

$$E(f(X_1)) \approx f(\mu_1)$$

$$\text{Var}(f(X_1)) \approx [f'(\mu_1)]^2\sigma^2_1$$

7  Exercise 3.4.13

The acceleration $g$ due to gravity is estimated by dropping an object and measuring the time it takes to travel a certain distance. Assume the distance $s$ is known to be exactly 5m, and the time is measured to be $t = 1.01 \pm 0.02$ s. Estimate $g$, and find the standard deviation of the estimate. (Note that $g = 2s/t^2$.)