Block sampler for univariate and multivariate asymmetric stochastic volatility models

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Time varying volatility models

Asset return

$$y_t = \log p_t - \log p_{t-1}$$

p_t: aseet price at time t. $\rightarrow$ variance varies over time.

- GARCH (Generalized Autoregressive Conditional Heterogeneity) model.

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- GARCH (Generalized Autoregressive Conditional Heterogeneity) model.
- EGARCH (Exponential GARCH) model.
- Stochastic Volatility model.

Univariate SV model

Stochastic Volatility model

- SV model

$$
\begin{aligned}
y_t &= \epsilon_t \exp(h_t/2), \quad \epsilon_t \sim \mathcal{N}(0,1), \\
h_{t+1} &= \mu + \phi h_t + \eta_t, \quad \eta_t \sim \mathcal{N}(0,\sigma_\eta^2),
\end{aligned}
\tag{1}
$$

Stochastic Volatility model

- SV model

$$\begin{aligned} y_t &= \epsilon_t \exp(h_t/2), \quad \epsilon_t \sim \mathcal{N}(0,1), \\ h_{t+1} &= \mu + \phi h_t + \eta_t, \quad \eta_t \sim \mathcal{N}(0,\sigma_\eta^2), \end{aligned} \tag{1}$$

- Asymmetry.

$$Corr(\epsilon_t, \eta_t) = \rho < 0, \tag{2}$$

Efficient Sampler for $\{h_t\}_{t=1}^n$

Efficient sampler for ASV models.

- Efficient sampler for SV model.
  - Block sampler: Shephard and Pitt (1997), Watanabe and Omori (2004). *Biometrika.*
  - 'Approximation' sampler (High efficiency).
    Kim, Shephard and Chib(1998). *Rev. Econ. Stud.*
    Chib, Nardari and Shephrad (2002). *J. Econometrics.*
- Efficient sampler for ASV model.
  - Block sampler: Omori and Watanabe (2008). *CSDA.*
  - 'Approximation' sampler:
    Omori, Chib, Shephard and Nakajima (2007). *J. Econometrics.*
    Nakajima and Omori (2008). *CSDA.*

Kim, Shephard and Chib (1998)

KSC (1998) considered SV model:

$$\begin{aligned} y_t &= \epsilon_t \exp(h_t/2), \\ h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t, \end{aligned}$$

where $\epsilon_t \sim N(0,1)$ and $\eta_t \sim N(0,\sigma^2)$. Transforming $y_t^* = \log y_t^2$, we obtain

$$\begin{aligned} y_t^* &= h_t + \xi_t, \\ h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t, \end{aligned}$$

where $\xi_t = \log \chi_1^2$. → Approximate by a normal mixture!

KSC (1998)

KSC approximate the pdf of ξ_t by v_t with a mixture of normal densities

$$g(v_t) = \sum_{i=1}^{K} p_i N(\mu_i, \sigma_i^2), \quad K = 7,$$

by numerical optimization.

- Match first four moments of $(\xi_t, v_t), (\exp(\xi_t), \exp(v_t))$
- Require $g(v_t)$ lies within a small distance of the true pdf of ξ_t.

Omori, Chib, Shephard and Nakajima (2007)

For ASV model, we consider the transformation:

$$y_t^* = \log(y_t^2), \quad \delta_t = I(y_t > 0) - I(y_t \leq 0),$$

so $y_t = \delta_t \exp(y_t^*/2)$. Since $\eta_t|\epsilon_t \sim N(\rho\sigma\epsilon_t, \sigma^2(1-\rho^2))$, we note that

$$h_{t+1}|\delta_t, \epsilon_t, h_t \sim N\left(\mu + \phi(h_t - \mu) + \delta_t\rho\sigma\exp\frac{\xi_t}{2}, \sigma^2(1-\rho^2)\right).$$

where $\xi_t = \log \epsilon_t^2 \quad (\exp(\xi_t/2) = |\epsilon_t|)$.

OCSN (2007)

s_t : a multinomial r.v. for the i-th component.
Approximating linear Gaussian state space model:

$$y_t^* = h_t + v_t, \quad h_{t+1} = \mu + \phi(h_t - \mu) + w_t,$$

(v_t, w_t) is a bivariate normal mixture. Given $s_t = i$ $(i = 1, \ldots, K)$

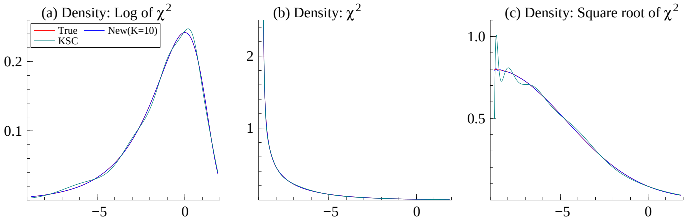
$$\begin{aligned} v_t &= \mu_i + \sigma_i z_t, \\ w_t &= \delta_t \rho\sigma (a_i + b_i z_t) \exp(\mu_i/2) + \sigma\sqrt{1-\rho^2} z_t^*, \end{aligned}$$

$z_t \sim N(0,1), z_t^* \sim N(0,1)$, and a_i, b_i are some constants.

OCSN (2007)

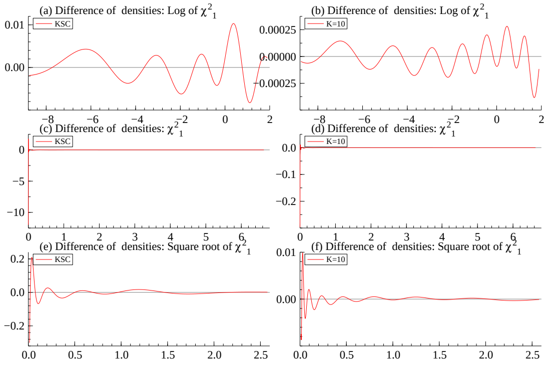
	KSC($K=7$)			$K=10$		
i	p_i	μ_i	σ_i^2	p_i	μ_i	σ_i^2
1	0.04395	1.50746	0.16735	0.00609	1.92677	0.11265
2	0.24566	0.52478	0.34023	0.04775	1.34744	0.17788
3	0.34001	−0.65098	0.64009	0.13057	0.73504	0.26768
4	0.25750	−2.35859	1.26261	0.20674	0.02266	0.40611
5	0.10556	−5.24321	2.61369	0.22715	−0.85173	0.62699
6	0.00002	−9.83726	5.17950	0.18842	−1.97278	0.98583
7	0.00730	−11.40039	5.79596	0.12047	−3.46788	1.57469
8				0.05591	−5.55246	2.54498
9				0.01575	−8.68384	4.16591
10				0.00116	−14.65000	7.33342

OCSN (2007)



Plot of $(v_t, \xi_t), (\exp(v_t), \exp(\xi_t)), (\exp(v_t/2), \exp(\xi_t/2))$.
$\rightarrow$ Approximation is improved.

OCSN (2007)



Difference between approximate and true densities.

OCSN (2007)

MCMC algorithm:

We sample in two blocks.
Let $\gamma = (\phi, \sigma^2, \rho)'$ and $\theta = (\mu, \gamma')'$.

1. Initialize $s = \{s_t\}_{t=1}^n$, $h = \{h_t\}_{t=1}^n$ and θ.
2. Sample $s|h, \theta, y^*, \delta$.
3. Sample $(h, \theta)|s, y^*, \delta$ as one block.
   3.1 Sample $\gamma|s, y^*, \delta$.
   3.2 Sample $\mu|\gamma, s, y^*, \delta$.
   3.3 Sample $h|\theta, s, y^*, \delta$.
4. Goto 2.

Use the integration sampler with the diffuse Kalman filter by de Jong (1991) (to intergate out μ).

Watanabe and Omori (2004)

Symmetric stochastic volatility model.

$$\begin{aligned} y_t &= \epsilon_t \sigma_\epsilon \exp(\alpha_t/2) \\ \alpha_{t+1} &= \phi \alpha_t + \eta_t \sigma_\eta \end{aligned}$$

$|\phi| < 1$. $\alpha_1 \sim N(0, \sigma_\eta^2/(1-\phi^2))$.

$$\begin{pmatrix} \epsilon_t \\ \eta_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right)$$

$\rightarrow$ To sample $\{h_t\}_{t=1}^n$, we construct a proposal density for sampling a block $h_t, \ldots, h_{t+k}$ given other h_s's and other parameters.

Watanabe and Omori (2004)

Taylor expansion of the log conditional posterior density:

$$
\begin{aligned}
&\log f(\eta_{t-1},\ldots,\eta_{t+k-1}|\alpha_{t-1},\alpha_{t+k+1},y_t,\ldots,y_{t+k}) \\
&\approx \quad \text{constant} - \frac{1}{2}\sum_{s=t-1}^{t+k-1}\eta_s^2 \\
&\quad + \sum_{s=t}^{t+k}\left[l(\hat{\alpha}_s) + (\alpha_s-\hat{\alpha}_s)l'(\hat{\alpha}_s) + \frac{1}{2}\{(\alpha_s-\hat{\alpha}_s)\}^2 l''(\hat{\alpha}_s)\right] \\
&\quad - \frac{1}{2\sigma_\eta^2}(\alpha_{t+k+1}-\phi\alpha_{t+k})^2 \equiv \log g \qquad (3)
\end{aligned}
$$

where $l(\alpha_s) = \log f(y_s|\alpha_s)$, $l'(\alpha_s) = \partial l(\alpha_s)/\partial\alpha_s$, and $l''(\alpha_s) = \partial^2 l(\alpha_s)/\partial\alpha_s^2$.

$\rightarrow$ Use g as a proposal for Metropolis-Hastings algorithm.

Watanabe and Omori (2004)

The posterior g corresponds to the state space model with the following measurement eq.

$$\hat{y}_s = \hat{\alpha}_s + v_s l'(\hat{\alpha}_s), \tag{4}$$

where

$$v_s = -1/l''(\hat{\alpha}_s), \tag{5}$$

and for $s = t + k < n$,

$$\hat{y}_s = v_s \left[\left\{ l'(\hat{\alpha}_s) - l''(\hat{\alpha}_s)\hat{\alpha}_s \right\} + \phi\sigma_\eta^{-2}\alpha_{s+1} \right], \tag{6}$$

where

$$v_s = \left[\phi^2\sigma_\eta^{-2} - l''(\hat{\alpha}_s) \right]^{-1}. \tag{7}$$

Omori and Watanabe (2008)

Asymmetric stochastic volatility model.

$$\begin{aligned} y_t &= \epsilon_t \sigma_\epsilon \exp(\alpha_t/2) \quad y_t\text{:stock return} \\ \alpha_{t+1} &= \phi \alpha_t + \eta_t \sigma_\eta \end{aligned}$$

$|\phi| < 1$. $\alpha_1 \sim N(0, \sigma_\eta^2/(1-\phi^2))$.

$$\begin{pmatrix} \epsilon_t \\ \eta_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right)$$

Approximation of the log conditional likelihood

$$
\begin{aligned}
y_t|\alpha_t, \alpha_{t+1} &\sim N(\mu_t, \sigma_t^2) \\
\mu_t &= \rho\sigma_\epsilon\sigma_\eta^{-1}(\alpha_{t+1} - \phi\alpha_t)\exp(\alpha_t/2), \quad t = 1, \ldots, n. \\
\sigma_t^2 &= \begin{cases} (1-\rho^2)\sigma_\epsilon^2 \exp(\alpha_t), & t = 1, \ldots, n-1, \\ \sigma_\epsilon^2 \exp(\alpha_n), & t = n, \end{cases}
\end{aligned}
$$

$\rightarrow$ Approximate $L = \sum_{s=t}^{t+k} l(\alpha_s, \alpha_{s+1}) - (\alpha_{s+k+1} - \phi\alpha_{s+k})^2/(2\sigma_\eta^2)$ using Taylor expansion around the conditional mode where $l(\alpha_s, \alpha_{s+1}) = \log f(y_s|\alpha_s, \alpha_{s+1})$.

Approximation of the log conditional likelihood

$$
\begin{aligned}
&\log f(\eta_s, \ldots, \eta_{s+k} | \alpha_s, \alpha_{s+k+1}, y_s, \ldots, y_{s+k+1}) \\
\approx \quad & \text{const} - \frac{1}{2}\sum_{t=s}^{s+k} \eta_t^2 + \hat{L} + \left.\frac{\partial L}{\partial \eta'}\right|_{\eta=\hat{\eta}} (\eta - \hat{\eta}) \\
& + \frac{1}{2}(\eta - \hat{\eta})' \, E\left(\frac{\partial L^2}{\partial \eta \partial \eta'}\right)\Big|_{\eta=\hat{\eta}} (\eta - \hat{\eta}) \\
= \quad & \text{const} + \log g(\eta_s, \ldots, \eta_{s+k} | \alpha_s, \alpha_{s+k+1}, y_s, \ldots, y_{s+k+1})
\end{aligned}
$$

$\rightarrow$ Construct the auxiliary state space model to obtain the sample from g (Omori and Watanabe (2008)).

Approximation of the log conditional likelihood

$$
\begin{aligned}
d &= \partial L/\partial \alpha|_{\eta=\hat{\eta}} \\
Q &= -E\left[\frac{\partial^2 L}{\partial\alpha\partial\alpha'}\right]_{\eta=\hat{\eta}} \\
&= \begin{pmatrix}
A_{s+1} & B'_{s+2} & O & \dots & O \\
B_{s+2} & A_{s+2} & B'_{s+3} & \dots & O \\
O & B_{s+3} & A_{s+3} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & B'_{s+m} \\
O & \dots & O & B_{s+m} & A_{s+m}
\end{pmatrix} \\
A_t &= -E\left[\frac{\partial^2 L}{\partial\alpha_t\partial\alpha'_t}\right]\Bigg|_{\eta=\hat{\eta}}, \quad B_t = -E\left[\frac{\partial^2 L}{\partial\alpha_t\partial\alpha'_{t-1}}\right]\Bigg|_{\eta=\hat{\eta}}
\end{aligned}
$$

Approximation of the log conditional likelihood

Calculate

- $D_{s+1} = A_{s+1}$
- $D_t = A_t - B_t D_{t-1}^{-1} B_t'$, $t = s+2, \ldots, s+m$
- K_t: Choleski decomposition of D_t $(D_t = K_t K_t')$.

Define auxiliary variable $\hat{y}_t = \hat{\gamma}_t + D_t^{-1} b_t$ $(s+1 \leq t \leq s+m)$ where

$$
\begin{aligned}
\hat{\gamma}_t &= \hat{\alpha}_t + K_t'^{-1} J_{t+1}' \hat{\alpha}_{t+1} \quad (\hat{\gamma}_{s+m} = \hat{\alpha}_{s+m}) \\
b_t &= d_t - J_t K_{t-1}^{-1} b_{t-1} \quad (b_{s+1} = d_{s+1}) \\
J_t &= B_t K_{t-1}^{-1'} \quad (J_{s+1} = O)
\end{aligned}
$$

Approximation of the log conditional likelihood

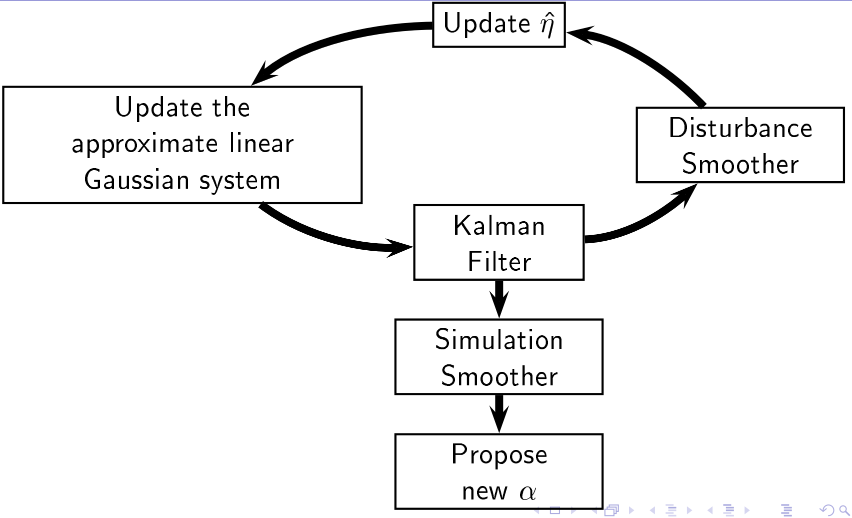
Approximate linear Gaussian system (← proposal)

$$\begin{aligned}
\alpha_{t+1} &= T_t \alpha_t + H_t \xi_t \\
\hat{y}_t &= Z_t \alpha_t + G_t \xi_t \\
\xi_t &= (\epsilon_t', \eta_t')' \sim N(0, I)
\end{aligned}$$

where

$$\begin{aligned}
Z_t &= I + K_t^{'-1} J_{t+1}' T_t \\
G_t &= K_t^{'-1} [I, J_{t+1}' R_t] \\
H_t &= [O, R_t] \quad (J_{s+m+1} = O)
\end{aligned}$$

Approximation of the log conditional likelihood



Generation of ϕ

$\pi(\phi)$: a prior pdf. The log conditional posterior density is

$$\text{const} + \log \pi(\phi) + \frac{1}{2}\log(1-\phi^2) - \frac{\alpha_1^2(1-\phi^2)}{2\sigma_\eta^2} - \frac{1}{2(1-\rho^2)\sigma_\eta^2}\sum_{t=1}^{n-1}\left(\alpha_{t+1} - \phi\alpha_t - \rho\sigma_\eta\sigma_\epsilon^{-1}e^{-\alpha_t/2}y_t\right)^2.$$

Proposal for MH algorithm; $\phi \sim TN_{(-1,1)}(\mu_\phi, \sigma_\phi^2)$ where

Generation of ϕ

$$\mu_\phi = \frac{\sum_{t=1}^{n-1} \alpha_t \left(\alpha_{t+1} - \rho\sigma_\eta\sigma_\epsilon^{-1} e^{-\alpha_t/2} y_t\right)}{\rho^2\alpha_1^2 + \sum_{t=2}^{n-1} \alpha_t^2},$$

$$\sigma_\phi^2 = \frac{(1-\rho^2)\sigma_\eta^2}{\rho^2\alpha_1^2 + \sum_{t=2}^{n-1} \alpha_t^2}.$$

Given the current sample ϕ_x, generate $\phi_y \sim TN_{(-1,1)}(\mu_\phi, \sigma_\phi^2)$ and accept it with probability

$$\min\left\{ \frac{\pi(\phi_y)\sqrt{1-\phi_y^2}}{\pi(\phi_x)\sqrt{1-\phi_x^2}}, 1 \right\}.$$

Sampling $(\rho, \sigma_\eta, \sigma_\epsilon)$

Define

$$\Sigma = \begin{pmatrix} \sigma_\epsilon^2 & \rho\sigma_\epsilon\sigma_\eta \\ \rho\sigma_\epsilon\sigma_\eta & \sigma_\eta^2 \end{pmatrix}.$$

Prior:($\Sigma^{-1} \sim W(\nu_0, \Sigma_0)$). The log conditional posterior for Σ is

$$\text{const} - \log\sigma_\eta - \frac{\alpha_1^2(1-\phi^2)}{2\sigma_\eta^2} - \frac{\nu_1}{2}\log|\Sigma| - \frac{1}{2}\text{tr}\left(\Sigma_1^{-1}\Sigma^{-1}\right)$$
$$- \log\sigma_\epsilon - \frac{y_n^2}{2\sigma_\epsilon^2\exp(\alpha_n)},$$

where $\nu_1 = \nu_0 + n - 1$, $\Sigma_1^{-1} = \Sigma_0^{-1} + \sum_{t=1}^{n-1} x_t x_t'$,
$x_t = (y_t e^{-\alpha_t/2}, \alpha_{t+1} - \phi\alpha_t)$.

Sampling $(\rho, \sigma_\eta, \sigma_\epsilon)$

MH algorithm using a proposal $\Sigma^{-1} \sim W(\nu_1, \Sigma_1)$.

Given the current value Σ_x^{-1}, generate $\Sigma_y^{-1} \sim W(\nu_1, \Sigma_1)$ and accept it with probability

$$\min\left\{ \frac{\sigma_{\epsilon,y}^{-1}\sigma_{\eta,y}^{-1} \exp - \frac{\alpha_1^2(1-\phi^2)}{2\sigma_{\eta,y}^2} - \frac{y_n^2}{2\sigma_{\epsilon,y}^2 \exp(\alpha_n)}}{\sigma_{\epsilon,x}^{-1}\sigma_{\eta,x}^{-1} \exp - \frac{\alpha_1^2(1-\phi^2)}{2\sigma_{\eta,x}^2} - \frac{y_n^2}{2\sigma_{\epsilon,x}^2 \exp(\alpha_n)}}, 1 \right\}.$$

Empirical study

TOPIX daily returns $y_t = 100 \times (\log p_t - \log p_{t-1})$. August 1, 1997 – July 31, 2002 (n=1230).

Param.	Mean	Stdev	95% interval	IF	CD
ϕ	0.945	0.018	[0.903, 0.974]	118.0	0.31
σ_ϵ	1.263	0.067	[1.133, 1.395]	30.0	0.22
σ_η	0.193	0.033	[0.136, 0.264]	209.9	0.29
ρ	-0.438	0.104	$[-0.632, -0.229]$	62.6	0.98

Table: Block sampler with $K = 40$. # iter: 75,000.

CD: p-value of the convergence diagnostic test by Geweke (1992).
IF: inefficiency factors.

Comparison of inefficiency factors

Param.	single move sampler	block sampler	mixture sampler
ϕ	2199.2	118.0	7.6
σ_ϵ	103.1	30.0	2.0
σ_η	3506.6	209.9	10.2
ρ	1038.0	62.6	4.8

Table: Single move sampler: Jacquier, Polson and Rossi (2004). J. Econometrics. # iter: 250,000 (single move sampler), 75,000 (block sampler), 5,000 (mixture sampler)

Inefficiency factors: $(1 + 2\sum_{s=1}^{\infty} \rho_s$, ρ_s ACF at lag s), (the numerical variance of the posterior sample mean) / (the variance of the posterior sample mean from the hypothetical uncorrelated draws).

Multivariate SV models

See an extensive survey:

Chib, S., Omori, Y. and Asai, M. (2008)
"Multivariate Stochastic Volatility,"
Handbook of Financial Time Series (eds T.G. Andersen, R.A. Davis, Jens-Peter Kreiss and T. Mikosch),
Springer-Verlag, New York, in press.

Factor MSV model

Mean factor model (factor & error: symmetric SV).

- Jacquier, Polson and Rossi (1999). error: constant covariance matrix. single move.
- Pitt and Shephard (1999). mixture sampler.
- Chib, Naradari and Shephard (2006). Jumps and heavy-tailed errors. mixture sampler.
- Lopes and Carvalho (2006). AR(1) for factor coefs. Markov switching in volatility level.
- Han (2006). AR(1) for factors.

Factor multivariate ASV model

$$\mathbf{y}_t = \mathbf{B}\mathbf{f}_t + \lambda_t^{-1/2}\mathbf{V}_{1t}^{1/2}\varepsilon_{1t}, \tag{8}$$

$$\mathbf{f}_t = \mathbf{V}_{2t}^{1/2}\varepsilon_{2t}, \tag{9}$$

$$\alpha_{t+1} = \mathbf{\Phi}\alpha_t + \eta_t, \tag{10}$$

$$\begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t \end{pmatrix} \sim \mathcal{N}_{2(p+q)}\left(\mathbf{0}, \begin{pmatrix} \Sigma_{\varepsilon\varepsilon} & \Sigma_{\varepsilon\eta} \\ \Sigma_{\varepsilon\eta} & \Sigma_{\eta\eta} \end{pmatrix}\right),$$

where $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}', \varepsilon_{2t}')'$ and

$$\mathbf{V}_t = \mathrm{diag}(\mathbf{V}_{1t}, \mathbf{V}_{2t}) = \mathrm{diag}(\exp(\alpha_{1t}), \ldots, \exp(\alpha_{p+q,t})), \tag{11}$$

$$\mathbf{\Phi} = \mathrm{diag}(\phi_1, \ldots, \phi_{p+q}) \tag{12}$$

Factor MASV model

$$\boldsymbol{\Sigma}_{\varepsilon\varepsilon} = \mathrm{diag}(\sigma_{1,\varepsilon\varepsilon}, \ldots, \sigma_{p+q,\varepsilon\varepsilon}) \tag{13}$$

$$\boldsymbol{\Sigma}_{\eta\eta} = \mathrm{diag}(\sigma_{1,\eta\eta}, \ldots, \sigma_{p+q,\eta\eta}) \tag{14}$$

$$\boldsymbol{\Sigma}_{\varepsilon\eta} = \mathrm{diag}\left(\rho_1 \sigma_{1,\varepsilon\varepsilon}^{1/2} \sigma_{1,\eta\eta}^{1/2}, \ldots, \rho_{p+q} \sigma_{p+q,\varepsilon\varepsilon}^{1/2} \sigma_{p+q,\eta\eta}^{1/2}\right), \tag{15}$$

and ($b_{ij} = 0$ for ($i < j$, $i \leq q$) and $b_{ii} = 1$ ($i \leq q$)).

*Note that we have conditionally univariate ASV models: $\rightarrow$ MCMC estimation based on the block sampler by Omori and Watanabe (2008).

Factor MASV model

Rewrite

$$\mathbf{w}_t = \begin{pmatrix} \mathbf{f}_t \\ \mathbf{y}_t \\ \boldsymbol{\alpha}_{t+1} - \boldsymbol{\Phi}\boldsymbol{\alpha}_t \end{pmatrix} = \begin{pmatrix} \mathbf{P}_{1t} \\ \mathbf{P}_{2t} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \eta_t \end{pmatrix},$$

$$\mathbf{P}_{1t} = (\mathbf{O}, \mathbf{V}_{2t}^{1/2}, \mathbf{O})$$

$$\mathbf{P}_{2t} = \begin{pmatrix} \lambda_t^{-1/2}\mathbf{V}_{1t}^{1/2}, & \mathbf{B}\mathbf{V}_{2t}^{1/2}, & \mathbf{O} \\ \mathbf{O}, & \mathbf{O}, & \mathbf{I} \end{pmatrix},$$

and consider the posterior density of β marginalized over $\mathbf{f}_t$.

MCMC estimation

1. Generate β. The full conditional distribution of β:

$$\pi(\beta|\phi, \alpha, \Sigma, \lambda, \mathbf{y})$$

$$\propto \pi(\beta) \prod_{t=1}^{n} |\mathbf{P}_{2t}\Sigma\mathbf{P}_{2t}'|^{-1/2} \times \exp -\frac{1}{2} \sum_{t=1}^{n} \tilde{\mathbf{w}}_t'(\mathbf{P}_{2t}\Sigma\mathbf{P}_{2t}')^{-1}\tilde{\mathbf{w}}_t$$

$$\tilde{\mathbf{w}}_t' = \left\{\mathbf{y}_t', (\alpha_{t+1} - \Phi\alpha_t)'\right\}$$

where $\pi(\beta)$ denotes a normal prior density. Propose a candidate $\beta_c \sim \mathcal{N}(\mu^*, \Sigma^*)$ and conduct MH algorithm where

$$\Sigma^* = \left\{ - \left. \frac{\partial^2 \log \pi(\beta|\cdot)}{\partial\beta\partial\beta'} \right|_{\beta=\hat{\beta}} \right\}^{-1},$$

$$\mu^* = \hat{\beta} + \Sigma^* \left. \frac{\partial \log \pi(\beta|\cdot)}{\partial\beta} \right|_{\beta=\hat{\beta}}$$

MCMC estimation

2. Generate $\{\mathbf{f}_t\}$.

$$\mathbf{f}_t | \boldsymbol{\beta}, \boldsymbol{\phi}, \boldsymbol{\alpha}, \boldsymbol{\Sigma}, \boldsymbol{\lambda}, \mathbf{y} \sim \mathcal{N}(\boldsymbol{\mu}_f, \boldsymbol{\Sigma}_f),$$
$$\boldsymbol{\Sigma}_f = \mathbf{P}_{1t}\boldsymbol{\Sigma}\mathbf{P}'_{1t} - \mathbf{P}_{1t}\boldsymbol{\Sigma}\mathbf{P}'_{2t}\left(\mathbf{P}_{2t}\boldsymbol{\Sigma}\mathbf{P}'_{2t}\right)^{-1}\mathbf{P}_{2t}\boldsymbol{\Sigma}\mathbf{P}'_{1t},$$
$$\boldsymbol{\mu}_f = \mathbf{P}_{1t}\boldsymbol{\Sigma}\mathbf{P}'_{2t}\left(\mathbf{P}_{2t}\boldsymbol{\Sigma}\mathbf{P}'_{2t}\right)^{-1}\tilde{\mathbf{w}}_t,$$

3. Generation $(\phi_j, \boldsymbol{\Sigma}_j, \boldsymbol{\alpha}_j)$ for $j = 1, \ldots, p+q$. Define $\mathbf{a}_t = \mathbf{B}\mathbf{f}_t$ and

$$y^*_{jt} = \begin{cases} \lambda_t^{1/2}\{y_{jt} - a_{jt}\}, & j = 1, \ldots, p, \\ f_{j-p,t}, & j = p+1, \ldots, p+q, \end{cases}$$

MCMC estimation

For $j = 1, \ldots, p+q$, we obtain conditionally independent ASV models:

$$\begin{aligned} y^*_{jt} &= \exp(\alpha_{jt}/2)\epsilon_{jt}, \\ \alpha_{j,t+1} &= \phi_j \alpha_{jt} + \eta_{jt} \end{aligned}$$

$$\mathbf{u}_{jt} = \begin{pmatrix} \epsilon_{jt} \\ \eta_{jt} \end{pmatrix} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_j\right), \quad j = 1, \ldots, p+q.$$

Using Omori and Watanabe (2008), we generate $(\phi_j, \boldsymbol{\Sigma}_j, \boldsymbol{\alpha}_j)$ for $j = 1, \ldots, p+q$.

MCMC estimation

4. Generate $\{\nu\}$.

$$\begin{aligned}\pi(\nu|\boldsymbol{\lambda}) &\propto \pi(\nu)\prod_{t=1}^{n}\pi(\lambda_t|\nu),\\ &\propto \pi(\nu)\left\{\frac{\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\right\}^{n}\left(\prod_{t=1}^{n}\lambda_t\right)^{\frac{\nu}{2}}\times\exp-\frac{\sum_{t=1}^{n}\lambda_t}{2}\nu\end{aligned}$$

where the prior $\nu \sim \mathcal{G}(m_0^\nu, S_0^\nu)$. Conduct MH algorithm using $\nu \sim \mathcal{TN}_{(0,\infty)}(\mu_\nu, \sigma_\nu^2)$ where

$$\sigma_\nu^2 = \left\{-\left.\frac{\partial^2 \log\pi(\nu|\boldsymbol{\lambda})}{\partial^2\nu}\right|_{\nu=\hat{\nu}}\right\}^{-1}, \quad \mu_\nu = \hat{\nu} + \sigma_\nu^2\left.\frac{\partial \log\pi(\nu|\boldsymbol{\lambda})}{\partial\nu}\right|_{\nu=\hat{\nu}}$$

($\hat{\nu}$:mode of $\pi(\nu|\boldsymbol{\lambda})$).

MCMC estimation

5. Generate $\{\lambda_t\}$ $(j = 1, \ldots, p, t = 1, \ldots, n)$.

$$\pi(\lambda_t | \beta, \mathbf{f}_t, \phi, \{\boldsymbol{\Sigma}_j\}_{j=1}^p, \boldsymbol{\alpha}_t, \nu, \mathbf{y})$$

$$\propto \begin{cases} \lambda_t^{\frac{\nu+p}{2}-1} \exp -\frac{1}{2}\left[\nu + \sum_{j=1}^p \frac{\{\tilde{y}_{jt} - c_{jt}\}^2}{\sigma_{j,\varepsilon\varepsilon}(1-\rho_j^2)\exp(\alpha_{jt})}\right]\lambda_t, & t < n, \\ \lambda_n^{\frac{\nu+p}{2}-1} \exp -\frac{1}{2}\left[\nu + \sum_{j=1}^p \frac{\tilde{y}_{jn}^2}{\sigma_{j,\varepsilon\varepsilon}\exp(\alpha_{jn})}\right]\lambda_n, & t = n, \end{cases}$$

$$c_{jt} = \rho_j \sqrt{\sigma_{j,\varepsilon\varepsilon}/(\sigma_{j,\eta\eta}\lambda_t)}(\alpha_{j,t+1} - \phi_j \alpha_{jt})\exp(\alpha_{jt}/2),$$

Sample using MH algorithm with a proposal

$$\mathcal{G}\left(\frac{\nu+p}{2}, \frac{1}{2}\left[\nu + \sum_{j=1}^p \frac{\tilde{y}_{jt}^2}{\sigma_{j,\varepsilon\varepsilon}(1-\rho_j^2)\exp(\alpha_{jt})}\right]\right).$$

Asymmetric MSV model

Multivariate Stochastic Volatility model

$$\mathbf{y}_t = \mathbf{V}_t^{1/2}\boldsymbol{\varepsilon}_t, \quad t = 1, \ldots, n, \tag{16}$$

$$\boldsymbol{\alpha}_{t+1} = \boldsymbol{\Phi}\boldsymbol{\alpha}_t + \boldsymbol{\eta}_t, \quad t = 1, \ldots, n-1, \tag{17}$$

$$\boldsymbol{\alpha}_1 \sim \mathcal{N}_p\left(\mathbf{0}, \boldsymbol{\Sigma}_0\right), \tag{18}$$

where $\boldsymbol{\Sigma}_0 = \boldsymbol{\Phi}\boldsymbol{\Sigma}_0\boldsymbol{\Phi} + \boldsymbol{\Sigma}_{\eta\eta}$, and

$$\begin{aligned} \mathbf{V}_t^{1/2} &= \mathrm{diag}\left(\exp(\alpha_{1t}/2), \ldots, \exp(\alpha_{pt}/2)\right) \\ \boldsymbol{\Phi} &= \mathrm{diag}(\phi_1, \ldots, \phi_p), \end{aligned}$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} |\boldsymbol{\alpha}_t \sim \mathcal{N}_{2p}(\mathbf{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{\varepsilon\varepsilon} & \boldsymbol{\Sigma}_{\varepsilon\eta} \\ \boldsymbol{\Sigma}_{\eta\varepsilon} & \boldsymbol{\Sigma}_{\eta\eta} \end{pmatrix}$$

Previous Literature

- Symmetric MSV.
  - Smith and Pitts (2006). $p = 2$. (simple) block sampler.
  - Bos and Shephard (2006). single move sampler.
- Asymmetric MSV.
  - Chan, Kohn and Kirby (2006). simple block sampler. Asymmetry using (Wong, Carter and Kohn (2003))

$$\boldsymbol{\Sigma}^{-1} = \mathbf{TGT}, \quad \mathbf{T} = \mathrm{diag}\left(\sqrt{G^{11}}, \ldots, \sqrt{G^{pp}}\right),$$

  - Ishihara and Omori (2008). single move vs multi-move samplers.

MCMC estimation

1. Generate $\alpha|\phi, \mathbf{\Sigma}, \mathbf{y}$.
   1.1 Generate K stochastic knots $(k_1, \ldots, k_K)$ and set $k_0 = 0, k_{K+1} = n$.
   1.2 Generate $\{\alpha_t\}_{t=k_{i-1}+1}^{k_i} | \{\alpha_t | t \leq k_{i-1}, t > k_i\}, \Phi, \mathbf{\Sigma}, Y_n$ for $i = 1, \ldots, K+1$
2. Generate $\mathbf{\Sigma}|\phi, \{\alpha_t\}_{t=1}^n, \mathbf{y}$.
3. Generate $\phi|\mathbf{\Sigma}, \{\alpha_t\}_{t=1}^n, \mathbf{y}$.

MCMC estimation (Block sampler for $\{\alpha_t\}$)

1. Generation of $\{\alpha_t\}_{t=1}^n$. Similar to Omori and Watanabe (2008), compute

$$\mathbf{A}_t = -E\left[\frac{\partial^2 L}{\partial\alpha_t\partial\alpha_t'}\right], \quad t = s+1,\ldots,s+m, \qquad (19)$$

$$\mathbf{B}_t = -E\left[\frac{\partial^2 L}{\partial\alpha_t\partial\alpha_{t-1}'}\right], \quad t = s+2,\ldots,s+m, \qquad (20)$$

$$\mathbf{B}_{s+1} = \mathbf{O}, \qquad (21)$$

and let $\mathbf{d}_t = \partial L/\partial\alpha_t'$ for $t = s+1,\ldots,s+m$.

MCMC estimation (Block sampler for $\{\alpha_t\}$)

First evaluate $\mathbf{d}_t, \mathbf{A}_t$ and $\mathbf{B}_t$ at the current mode, $\boldsymbol{\alpha} = \hat{\boldsymbol{\alpha}}$, and compute as in Omori and Watanabe (2008) to obtain the approximating state space model:

$$\begin{aligned} \hat{\mathbf{y}}_t &= \mathbf{Z}_t \boldsymbol{\alpha}_t + \mathbf{G}_t \boldsymbol{\xi}_t, \quad t = s+1, \ldots, s+m, \\ \boldsymbol{\alpha}_{t+1} &= \boldsymbol{\Phi} \boldsymbol{\alpha}_t + \mathbf{H}_t \boldsymbol{\xi}_t, \quad t = s+1, \ldots, s+m-1, \\ \boldsymbol{\xi}_t &= (\boldsymbol{\epsilon}_t', \boldsymbol{\eta}_t')' \sim \mathcal{N}_{2p}(\mathbf{0}, \mathbf{I}), \end{aligned}$$

Repeat the disturbance smoother (Koopman (1993)) several times to update the value of $\hat{\boldsymbol{\alpha}}$. Use this state space model to generate a proposal of $\boldsymbol{\eta}_t$'s, using a simulation smoother by de Jong and Shephard (1995) or Durbin and Koopman (2002).

MCMC estimation (Single move sampler for $\{\alpha_t\}$)

Generate $\alpha_t | \{\alpha_s\}_{s \neq t}, \phi, \Sigma, Y_n$: using MH algorithm. Propose a candidate $\alpha_t^{\dagger}$ from $\alpha_t^{\dagger} \sim \mathrm{N}(\mathbf{m}_{\alpha_t}, \Sigma_{\alpha_t})$ and accept it with probability

$$\min\left\{\exp\{g(\alpha_t^{\dagger}) - g(\alpha_t)\}, 1\right\}$$

for $t = 1, \ldots, n$ where $g(\alpha_t) = -\frac{1}{2}\mathbf{y}_t'\Sigma_t^{-1}\mathbf{y}_t + \mathbf{y}_t'\Sigma_t^{-1}\mu_t$, and $\mu_t = \mathbf{V}_t^{1/2}\mathbf{m}_t$, $\Sigma_t = \mathbf{V}_t^{1/2}\mathbf{S}_t\mathbf{V}_t^{1/2}$ and

$$\mathbf{m}_t = \begin{cases} \Sigma_{\epsilon\eta}\Sigma_{\eta\eta}^{-1}(\alpha_{t+1} - \Phi\alpha_t), & t < n, \\ \mathbf{0} & t = n, \end{cases}$$

$$\mathbf{S}_t = \begin{cases} \Sigma_{\epsilon\epsilon} - \Sigma_{\epsilon\eta}\Sigma_{\eta\eta}^{-1}\Sigma_{\eta\epsilon}, & t < n, \\ \Sigma_{\epsilon\epsilon} & t = n. \end{cases}$$

(Definitions of $\mathbf{m}_{\alpha_t}, \Sigma_{\alpha_t}$ are omitted)

MCMC estimation (Generation of Σ)

2. Generate of Σ using MH algorithm. Propose a candidate $\mathbf{\Sigma}^{\dagger} \sim \mathcal{IW}(n_1, \mathbf{R}_1)$ and accept it with probability

$$\min\{g(\mathbf{\Sigma}^{\dagger})/g(\mathbf{\Sigma}), 1\}$$

where $n_1 = n_0 + n - 1$, $\mathbf{R}_1^{-1} = \mathbf{R}_0^{-1} + \sum_{t=1}^{n-1} \mathbf{v}_t \mathbf{v}_t'$ and

$$\begin{aligned} g(\mathbf{\Sigma}) &= |\mathbf{\Sigma}_0|^{-\frac{1}{2}} |\mathbf{\Sigma}_{\epsilon\epsilon}|^{-\frac{1}{2}} \\ &\quad \times \exp\left\{-\frac{1}{2}\left(\boldsymbol{\alpha}_1'\mathbf{\Sigma}_0^{-1}\boldsymbol{\alpha}_1 + \mathbf{y}_n'\mathbf{V}_n^{-1/2}\mathbf{\Sigma}_{\epsilon\epsilon}^{-1}\mathbf{V}_n^{-1/2}\mathbf{y}_n\right)\right\}, \\ \mathbf{v}_t &= \begin{pmatrix} \mathbf{V}_t^{-1/2}\mathbf{y}_t \\ \boldsymbol{\alpha}_{t+1} - \mathbf{\Phi}\boldsymbol{\alpha}_t \end{pmatrix}. \end{aligned}$$

MCMC estimation (Generation of ϕ)

3 Generate of ϕ using MH algorithm. Propose a candidate

$$\phi^{\dagger} \sim \mathcal{TN}_R(\boldsymbol{\mu}_{\phi}, \boldsymbol{\Sigma}_{\phi}), \quad R = \{\phi : |\phi_j| < 1, j = 1, \ldots, p\},$$
$$\boldsymbol{\mu}_{\phi} = \boldsymbol{\Sigma}_{\phi}\mathbf{b}, \quad \boldsymbol{\Sigma}_{\phi}^{-1} = \boldsymbol{\Sigma}^{22} \odot \mathbf{A},$$
$$\mathbf{A} = \sum_{t=1}^{n-1} \boldsymbol{\alpha}_t \boldsymbol{\alpha}_t', \quad \mathbf{B} = \sum_{t=1}^{n-1} \{\boldsymbol{\alpha}_t \mathbf{y}_t' \mathbf{V}_t^{-1/2} \boldsymbol{\Sigma}^{12} + \boldsymbol{\alpha}_t \boldsymbol{\alpha}_{t+1}' \boldsymbol{\Sigma}^{22}\},$$

$(b_i = B_{ii})$ and accept it with probability $\min\{h(\phi^{\dagger})/h(\phi), 1\}$ where

$$h(\phi) = |\boldsymbol{\Sigma}_0|^{-\frac{1}{2}} \prod_{j=1}^{p} (1+\phi_j)^{a_j - 1} (1-\phi_j)^{b_j - 1} \exp\left\{-\frac{1}{2}\boldsymbol{\alpha}_1' \boldsymbol{\Sigma}_0^{-1} \boldsymbol{\alpha}_1\right\}.$$

Illustrative examples: Single vs Block sampler

Table: Single move sampler (n=3000. # iter:200,000.)

Param.	True	Mean	(Stdev)	IF
ϕ_1	0.97	0.962	(0.007)	321.8
ϕ_2	0.97	0.959	(0.008)	243.7
$\sqrt{\sigma_{1,\epsilon\epsilon}}$	1.2	1.126	(0.054)	2698.8
$\sqrt{\sigma_{2,\epsilon\epsilon}}$	1.2	1.203	(0.061)	1916.7
$\sqrt{\sigma_{1,\eta\eta}}$	0.2	0.204	(0.018)	481.0
$\sqrt{\sigma_{2,\eta\eta}}$	0.2	0.223	(0.020)	259.8
$\rho_{12,\epsilon\epsilon}$	0.6	0.602	(0.012)	16.0
$\rho_{12,\eta\eta}$	0.7	0.628	(0.066)	399.8
$\rho_{11,\epsilon\eta}$	-0.2	-0.298	(0.063)	382.9
$\rho_{22,\epsilon\eta}$	-0.2	-0.228	(0.061)	108.1
$\rho_{12,\epsilon\eta}$	-0.1	-0.125	(0.063)	149.1
$\rho_{21,\epsilon\eta}$	-0.1	-0.178	(0.065)	301.3

Illustrative examples: Single vs Block sampler

Table: Block move sampler $K = 100$. (n=3000, # iter:50,000).

Param.	True	Mean	(Stdev)	IF
ϕ_1	0.97	0.962	(0.007)	128.0
ϕ_2	0.97	0.960	(0.007)	148.8
$\sqrt{\sigma_{1,\epsilon\epsilon}}$	1.2	1.143	(0.056)	106.4
$\sqrt{\sigma_{2,\epsilon\epsilon}}$	1.2	1.215	(0.062)	183.1
$\sqrt{\sigma_{1,\eta\eta}}$	0.2	0.201	(0.018)	197.0
$\sqrt{\sigma_{2,\eta\eta}}$	0.2	0.219	(0.018)	141.6
$\rho_{12,\epsilon\epsilon}$	0.6	0.602	(0.012)	8.0
$\rho_{12,\eta\eta}$	0.7	0.632	(0.065)	113.3
$\rho_{11,\epsilon\eta}$	-0.2	-0.298	(0.064)	79.8
$\rho_{22,\epsilon\eta}$	-0.2	-0.224	(0.062)	59.4
$\rho_{12,\epsilon\eta}$	-0.1	-0.121	(0.065)	74.9
$\rho_{21,\epsilon\eta}$	-0.1	-0.178	(0.066)	32.9

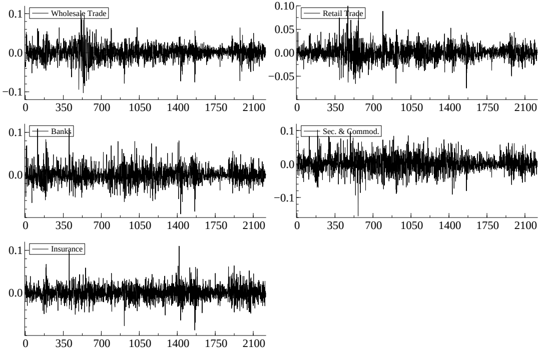
Empirical study

Tokyo Stock Exchange Indices.

- Five stock price indices from 33 industry sectors.
- Wholesale Trade, Retail Trade, Banks, Securties & Commodity Futures, Insurance.
- January 4, 1998 – December 29, 2006.

Five stock price indices

Figure: Plot of five stock price indices.



Empirical study. MASV model

Param.	Mean	(Stdev)	IF
ϕ_1	0.985	(0.004)	793.5
ϕ_2	0.984	(0.004)	819.0
ϕ_3	0.974	(0.007)	793.0
ϕ_4	0.982	(0.005)	727.6
ϕ_5	0.968	(0.008)	661.5

Table: Estimates of ϕ.

Empirical study. MASV model

Param.	Mean	(Stdev)	IF
$\sqrt{\sigma_{1,\epsilon\epsilon}}$	1.655	(0.237)	1315.2
$\sqrt{\sigma_{2,\epsilon\epsilon}}$	1.400	(0.196)	1259.2
$\sqrt{\sigma_{3,\epsilon\epsilon}}$	1.744	(0.167)	1034.3
$\sqrt{\sigma_{4,\epsilon\epsilon}}$	2.308	(0.276)	1113.2
$\sqrt{\sigma_{5,\epsilon\epsilon}}$	1.618	(0.111)	916.6
$\sqrt{\sigma_{1,\eta\eta}}$	0.202	(0.017)	279.7
$\sqrt{\sigma_{2,\eta\eta}}$	0.207	(0.018)	338.4
$\sqrt{\sigma_{3,\eta\eta}}$	0.234	(0.022)	323.0
$\sqrt{\sigma_{4,\eta\eta}}$	0.192	(0.018)	241.1
$\sqrt{\sigma_{5,\eta\eta}}$	0.199	(0.020)	363.3

Table: Estimates of $\boldsymbol{\Sigma}$.

Empirical study. MASV model

Param.	Mean	(Stdev)	IF
$\rho_{11,\epsilon\eta}$	-0.236	(0.054)	98.7
$\rho_{22,\epsilon\eta}$	-0.155	(0.059)	54.9
$\rho_{33,\epsilon\eta}$	-0.168	(0.057)	40.0
$\rho_{44,\epsilon\eta}$	-0.231	(0.058)	83.8
$\rho_{55,\epsilon\eta}$	-0.105	(0.072)	50.4

Table: Estimates of $\boldsymbol{\Sigma}$.

Empirical study. MASV model

Param.	Mean	(Stdev)	IF
$\rho_{12,\epsilon\epsilon}$	0.699	(0.012)	7.7
$\rho_{13,\epsilon\epsilon}$	0.673	(0.012)	21.5
$\rho_{14,\epsilon\epsilon}$	0.727	(0.011)	13.6
$\rho_{15,\epsilon\epsilon}$	0.547	(0.016)	11.6
$\rho_{23,\epsilon\epsilon}$	0.651	(0.013)	23.7
$\rho_{24,\epsilon\epsilon}$	0.665	(0.013)	6.8
$\rho_{25,\epsilon\epsilon}$	0.589	(0.015)	6.5
$\rho_{34,\epsilon\epsilon}$	0.717	(0.011)	15.3
$\rho_{24,\epsilon\epsilon}$	0.621	(0.014)	20.3
$\rho_{45,\epsilon\epsilon}$	0.558	(0.016)	7.7

Table: Estimates of Σ.

Empirical study. MASV model

Param.	Mean	(Stdev)	IF
$\rho_{12,\eta\eta}$	0.919	(0.030)	675.4
$\rho_{13,\eta\eta}$	0.816	(0.051)	545.9
$\rho_{14,\eta\eta}$	0.910	(0.031)	408.9
$\rho_{15,\eta\eta}$	0.850	(0.050)	879.5
$\rho_{23,\eta\eta}$	0.800	(0.059)	436.9
$\rho_{24,\eta\eta}$	0.850	(0.049)	304.8
$\rho_{25,\eta\eta}$	0.807	(0.060)	568.1
$\rho_{34,\eta\eta}$	0.851	(0.045)	350.4
$\rho_{24,\eta\eta}$	0.804	(0.056)	634.2
$\rho_{45,\eta\eta}$	0.803	(0.063)	758.7

Table: Estimates of $\boldsymbol{\Sigma}$.

Empirical study. MASV model

Param.	Mean	(Stdev)	IF	Param.	Mean	(Stdev)	IF
$\rho_{12,\epsilon\eta}$	-0.177	(0.058)	36.6	$\rho_{34,\epsilon\eta}$	-0.129	(0.063)	92.9
$\rho_{13,\epsilon\eta}$	-0.229	(0.055)	28.0	$\rho_{35,\epsilon\eta}$	0.044	(0.067)	81.7
$\rho_{14,\epsilon\eta}$	-0.247	(0.057)	69.4	$\rho_{41,\epsilon\eta}$	-0.204	(0.061)	94.8
$\rho_{15,\epsilon\eta}$	-0.122	(0.061)	76.9	$\rho_{42,\epsilon\eta}$	-0.156	(0.062)	44.6
$\rho_{21,\epsilon\eta}$	-0.240	(0.057)	84.8	$\rho_{43,\epsilon\eta}$	-0.177	(0.060)	57.3
$\rho_{23,\epsilon\eta}$	-0.166	(0.058)	60.6	$\rho_{45,\epsilon\eta}$	-0.071	(0.069)	178.9
$\rho_{24,\epsilon\eta}$	-0.215	(0.060)	51.0	$\rho_{51,\epsilon\eta}$	-0.208	(0.070)	184.4
$\rho_{25,\epsilon\eta}$	-0.108	(0.065)	74.0	$\rho_{52,\epsilon\eta}$	-0.079	(0.072)	78.6
$\rho_{31,\epsilon\eta}$	-0.130	(0.062)	97.7	$\rho_{53,\epsilon\eta}$	-0.139	(0.070)	110.9
$\rho_{32,\epsilon\eta}$	-0.077	(0.063)	45.2	$\rho_{54,\epsilon\eta}$	-0.187	(0.076)	176.4

Table: Estimates of Σ.

Summary

- Efficient sampler for ASV, Multivariate factor ASV and Multivariate ASV models.
- Comparison with single move samplers.
- Illustrative examples and empirical study.
- Future work: Model comparison using a marginal likelihood and Value at Risk.

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