DISCUSSION 2

1 Some Property

1. $P(A^c) = 1 - P(A)$
2. $P(\emptyset) = 0$
3. If $A \subset B$, then $P(A) \leq P(B)$
4. Addition Law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Especially, if $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$.
5. $(A \cap B)^c = A^c \cup B^c$, or $(A \cup B)^c = A^c \cap B^c$.
6. $A \cup B = (A \cap B^c) \cup B$.
7. $A = (A \cap B) \cup (A \cap B^c)$.

2 Counting Methods for Computing Probabilities

1. Uniform probability model: If $S = \{w_1, w_2, \ldots, w_n\}, P(w_1) = P(w_2) = \ldots = P(w_n) = \frac{1}{n}$, then the probability of A is $P(A) = \frac{\#\, of\, A}{\#\, of\, S}$.
2. Multiplication Priciple:
   If one experiment has m outcomes and another experiment has n outcomes, then there are mn possible outcomes for the two experiments.
3. Permutation
   Permutation is an ordered arrangement of outcomes. Suppose we have n objects and choose r objects and list them in order,
   with replacement: n^r
   without replacement:
   $$\frac{n!}{(n-r)!} = n(n-1)(n-2)...(n-r+1)$$
4. Combination
   If r objects are taken from n objects without replacement and disregarding the order,
   $$\binom{n}{r} == \frac{n!}{(n-r)!r!} = \frac{n \times (n-1) \times \ldots \times (n-r+1)}{r \times (r-1) \times \ldots \times 1}$$
   Properities
   (a) $\binom{n}{r} = \binom{n}{n-r}$
   (b) $\binom{n}{n} = \binom{n}{0} = 1$
   (c) $\binom{n}{1} = \binom{n}{n-1} = n$