33. 

a. The boxplot indicates a very slight positive skew, with no outliers. The data appears to center near 438.

b. Based on a normal probability plot, it is reasonable to assume the sample observations came from a normal distribution.

c. With d.f. = n − 1 = 16, the critical value for a 95% C.I. is $t_{0.05,16} = 2.120$, and the interval is $438.29 \pm (2.120 \left( \frac{15.14}{\sqrt{17}} \right)) = 438.29 \pm 7.785 = (430.51, 446.08)$. Since 440 is within the interval, 440 is a plausible value for the true mean. 450, however, is not, since it lies outside the interval.

36. 

$n = 26$, $\bar{x} = 370.69$, $s = 24.36$; $t_{0.05,25} = 1.708$

a. A 95% upper confidence bound: $370.69 + (1.708 \left( \frac{24.36}{\sqrt{26}} \right)) = 370.69 + 8.16 = 378.85$

b. A 95% upper prediction bound:

$370.69 + 1.708(24.36) \sqrt{1 + \frac{1}{26}} = 370.69 + 42.45 = 413.14$
c. Following a similar argument as that on p. 300 of the text, we need to find the variance of \( X - X_{\text{new}} \): 
\[
V(X - X_{\text{new}}) = V(X) + V(X_{\text{new}}) = V(X) + V(\frac{1}{2}(X_{27} + X_{28}))
\]
\[
= V(X) + V(\frac{1}{2}X_{27}) + V(\frac{1}{2}X_{28}) = V(X) + \frac{1}{4}V(X_{27}) + \frac{1}{4}V(X_{28})
\]
\[
= \frac{\sigma^2}{n} + \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2 = \sigma^2\left(\frac{1}{2} + \frac{1}{n}\right)
\]
We eventually arrive at \( T = \frac{X - X_{\text{new}}}{s \sqrt{\frac{1}{2} + \frac{1}{n}}} \sim t \)
distribution with \( n - 1 \) d.f., so the new prediction interval is \( x \pm t_{\alpha/2, n-1} \cdot s \sqrt{\frac{1}{2} + \frac{1}{n}} \). For this situation, we have
\[
370.69 \pm 2.305 \cdot 30.53 = 370.69 \pm 70.53 = (300.16, 441.22)
\]

46.

a. Using a normal probability plot, we ascertain that it is plausible that this sample was taken from a normal population distribution. (see the R code and plot on page 3).

b. With \( s = 1.579 \), \( n = 15 \), and \( X_{92,14}^2 = 6.571 \) the 95% upper confidence bound for \( \sigma \) is
\[
\frac{14(1.579)^2}{6.571} = 2.305
\]

60.

The length of the interval is \( (z_\gamma + z_{\alpha-\gamma}) \frac{s}{\sqrt{n}} \), which is minimized when \( z_\gamma + z_{\alpha-\gamma} \) is minimized, i.e. when \( \Phi^{-1}(1-\gamma) + \Phi^{-1}(1-\alpha+\gamma) \) is minimized. Taking \( \frac{d}{d\gamma} \) and equating to 0 yields
\[
\frac{1}{\Phi(1-\gamma)} = \frac{1}{\Phi(1-\alpha+\gamma)} \quad \text{where } \Phi(\bullet) \text{ is the standard normal p.d.f., whence}
\]
\[
\gamma = \frac{\alpha}{2}.
\]
Problem 46

R code:

```r
x <- c(26.7, 25.8, 24.0, 24.9, 26.4, 25.9, 24.4, 21.7, 24.1, 25.9, 27.3, 26.9, 27.3, 24.8, 23.6)
norm <- Norm(x)
qqline(x)
```

Then the plot will be:

![Normal Q-Q Plot](image)

You can also try `qqmath`.