3. Suppose that a random sample of 50 bottles of a particular brand of cough syrup is selected and the alcohol content of each bottle is determined. Let \( \mu \) denote the average alcohol content for the population of all bottles of the brand under study. Suppose that the resulting 95% confidence interval is (7.8, 9.4).

   a. Would a 90% confidence interval calculated from this same sample have been narrower or wider than the given interval? Explain your reasoning.
   b. Consider the following statement: There is a 95% chance that \( \mu \) is between 7.8 and 9.4. Is this statement correct? Why or why not?
   c. Consider the following statement: We can be highly confident that 95% of all bottles of this type of cough syrup have an alcohol content that is between 7.8 and 9.4. Is this statement correct? Why or why not?
   d. Consider the following statement: If the process of selecting a sample of size 50 and then computing the corresponding 95% interval is repeated 100 times, 95 of the resulting intervals will include \( \mu \). Is this statement correct? Why or why not?

4. A CI is desired for the true average stray-load loss \( \mu \) (watts) for a certain type of induction motor when the line current is held at 10 amps for a speed of 1500 rpm. Assume that stray-load loss is normally distributed with \( \sigma = 3.0 \).

   a. Compute a 95% CI for \( \mu \) when \( n = 25 \) and \( \bar{x} = 58.3 \).
   b. Compute a 95% CI for \( \mu \) when \( n = 100 \) and \( \bar{x} = 58.3 \).
   c. Compute a 99% CI for \( \mu \) when \( n = 100 \) and \( \bar{x} = 58.3 \).
   d. Compute an 82% CI for \( \mu \) when \( n = 100 \) and \( \bar{x} = 58.3 \).
   e. How large must \( n \) be if the width of the 99% interval for \( \mu \) is to be 1.0?

8. Let \( \alpha_1 > 0, \alpha_2 > 0 \), with \( \alpha_1 + \alpha_2 = \alpha \). Then

\[
P\left( -z_{\alpha_1} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < z_{\alpha_2} \right) = 1 - \alpha
\]

   a. Use this equation to derive a more general expression for a \( 100(1 - \alpha)% \) CI for \( \mu \) of which the interval (7.5) is a special case.
   b. Let \( \alpha = .05 \) and \( \alpha_1 = \alpha/4, \alpha_2 = 3\alpha/4 \). Does this result in a narrower or wider interval than the interval (7.5)?

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*The article “Evaluating Tunnel Kiln Performance” (Amer. Ceramic Soc. Bull., Aug. 1997: 59–63) gave the following summary information for fracture strengths (MPa) of \( x \) = 169 ceramic bars fired in a particular kiln: \( \bar{x} = 89.10, s = 3.73 \).

   a. Calculate a (two-sided) confidence interval for true average fracture strength using a confidence level of 95%. Does it appear that true average fracture strength has been precisely estimated?
   b. Suppose the investigators had believed a priori that the population standard deviation was about 4 MPa. Based on this supposition, how large a sample would have been required to estimate \( \mu \) to within .5 MPa with 95% confidence?
The Associated Press (October 9, 2002) reported that in a survey of 4722 American youngsters aged 6 to 19, 15% were seriously overweight (a body mass index of at least 30; this index is a measure of weight relative to height). Calculate and interpret a confidence interval using a 99% confidence level for the proportion of all American youngsters who are seriously overweight.