2. When an automobile is stopped by a roving safety patrol, each tire is checked for tire wear, and each headlight is checked to see whether it is properly aimed. Let $X$ denote the number of headlights that need adjustment, and let $Y$ denote the number of defective tires.

a. If $X$ and $Y$ are independent with $p_X(0) = .5$, $p_X(1) = .3$, $p_X(2) = .2$, and $p_X(0) = .6$, $p_Y(1) = .1$, $p_Y(2) = p_Y(3) = .05$, $p_Y(4) = .2$, display the joint pmf of $(X,Y)$ in a joint probability table.

b. Compute $P(X \leq 1$ and $Y \leq 1$) from the joint probability table, and verify that it equals the product $P(X \leq 1) \cdot P(Y \leq 1)$.

c. What is $P(X + Y = 0)$ (the probability of no violations)?

d. Compute $P(X + Y \leq 1)$.

8. A batchroom currently has 30 components of a certain type, of which 8 were provided by supplier 1, 10 by supplier 2, and 12 by supplier 3. Six of these are to be randomly selected for a particular assembly. Let $X =$ the number of supplier 1's components selected, $Y =$ the number of supplier 2's components selected, and $p(x, y)$ denote the joint pmf of $X$ and $Y$.

a. What is $p(3, 2)$? 

[Hint: Each sample of size 6 is equally likely to be selected. Therefore, $p(3, 2) =$ (number of outcomes with $X = 3$ and $Y = 2$)/(total number of outcomes). Now use the product rule for counting to obtain the numerator and denominator.]

b. Using the logic of part (a), obtain $p(x, y)$. (This can be thought of as a multivariate hypergeometric distribution—sampling without replacement from a finite population consisting of more than two categories.)

10. Annie and Alvie have agreed to meet between 5:00 P.M. and 6:00 P.M. for dinner at a local health-food restaurant. Let $X =$ Annie's arrival time and $Y =$ Alvie's arrival time. Suppose $X$ and $Y$ are independent with each uniformly distributed on the interval $[5, 6]$.

a. What is the joint pdf of $X$ and $Y$?

b. What is the probability that they both arrive between 5:15 and 5:45?

c. If the first one to arrive will wait only 10 min before leaving to eat elsewhere, what is the probability that they have dinner at the health-food restaurant? 

[Hint: The event of interest is $A =$ $\{(x, y); |x - y| \leq \frac{1}{3}\}$.

12. Two components of a minicomputer have the following joint pdf for their useful lifetimes $X$ and $Y$:

$$f(x, y) = \begin{cases} xe^{-x(1+y)} & x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

a. What is the probability that the lifetime $X$ of the first component exceeds 3?

b. What are the marginal pdf's of $X$ and $Y$? Are the two lifetimes independent? Explain.

c. What is the probability that the lifetime of at least one component exceeds 3?