1 Review

1.1 Continuous Random Variable

- Probability density function for continuous r.v.
  - Total area under the curve should be 1.
  - \( P(a \leq X \leq b) = \) area under the curve between \( a \) and \( b \).
  - \( f(x) \geq 0 \) for all \( x \).

- For continuous r.v. \( P(X = x) = 0 \), so
  \[
P(a \leq X \leq b) = P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)
  \]

1.2 Normal Distribution

- Normal Distribution
  - Normal Distribution: \( X \sim N(\mu, \sigma) \).
    - Bell-shaped and symmetric density with mean \( \mu \) and standard deviation \( \sigma \).
  - Standard Normal Distribution: \( N(0,1) \).

- Z-transformation
  If \( X \) is distributed as \( N(\mu, \sigma) \), then the standardized variable

  \[
  Z = \frac{X - \mu}{\sigma} \sim N(0,1)
  \]

  \[
P(a \leq X \leq b) = P\left( \frac{a - \mu}{\sigma} \leq Z \leq \frac{b - \mu}{\sigma} \right)
  \]
1.3 The Normal Approximation to the Binomial

- **Review**: If $X \sim Bin(n, p)$, then $E(X) = np$, $Var(X) = np(1 - p)$, hence $sd(X) = \sqrt{np(1 - p)}$.

- **Normal Approximation to Binomial**
  When $X \sim Bin(n, p)$ and $np \geq 15$, $n(1 - p) \geq 15$, then $X \approx N(np, \sqrt{np(1 - p)})$. Therefore,
  
  \[ Z = \frac{X - np}{\sqrt{np(1 - p)}} \] 

  is approximately $N(0, 1)$.

2 Examples

1. For a standard normal r.v. $Z$, find
   
   (a) $P(Z < .14)$
   
   (b) The 14th percentile of the standard normal distribution. That is, find the $z$ that satisfies $P(Z \leq z) = .14$.
   
   (c) $P(Z < .86)$
   
   (d) The 86th percentile of the standard normal distribution.

2. If $X \sim N(80, 4)$, find
   
   (a) $P(X > 71)$
   
   (b) $P(73 \leq X \leq 89)$

3. If $X \sim N(150, 5)$, find $b$ such that
   
   (a) $P(X > b) = .025$
   
   (b) $P(X < b) = .305$

4. The diameter of hail hitting the ground during a storm is normally distributed with a mean of 0.5 inch and a standard deviation of 0.1 inch. What is the probability that
   
   (a) A hailstorm picked up at random will have a diameter greater than 0.75 inch?
   
   (b) Two hailstorms picked up in a row will have diameters greater than 0.6 inch when we assume independence of two diameters?
   
   (c) By the end of the storm, what proportion of the hailstorms would have had diameters greater than 0.75 inch?

5. The unemployment rate in a city is 7.9%. A sample of 300 persons is selected from the labor force. Approximate the probability that
   
   (a) Fewer than 18 unemployed persons are in the sample.
   
   (b) More than 30 unemployed persons are in the sample.