Discussion 3

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1 TA information

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2 Review

2.1 Sample Space and Events

• Experiment: the process of observing a phenomenon that has variation in its outcomes.

• Sample Space: the collection of all possible distinct outcomes of the experiment.

• Event: a subset of a sample space.

  – The probability of an event is a numerical value that represents the proportion of times the event is expected to occur when the experiment is repeated under identical conditions.
  – The probability of event A is denoted by $P(A)$.

• Basic Concepts in Set Theory

  – The union of A and B, denoted by $A \cup B$, is the event consisting of all outcomes that are either in A or B or in both events.
  – The intersection of A and B, denoted by $AB$, is the event consisting of all outcomes that are in both A and B.
  – The complement of A, denoted by $\bar{A}$, is the set of all outcomes that are not contained in A.
  – Two events A and B are mutually exclusive when $P(AB) = 0$.

2.2 Assigning Probability

• When the elementary outcomes are modeled as equally likely, we have a uniform probability model. If there are $k$ elementary outcomes in $S$, each is assigned the probability of $1/k$.

• An event $A$ consisting of $m$ elementary outcomes is then assigned

$$P(A) = \frac{m}{k} = \frac{\text{Number of elementary outcomes in } A}{\text{Number of elementary outcomes in } S}$$
2.3 Some Basic Rules

- **Law of Complement**: \( P(\overline{A}) = 1 - P(A) \)
- **Addition Law**: \( P(A \text{ or } B) = P(A) + P(B) - P(AB) \)
- **Addition Law for Mutually Exclusive Events**: \( P(A \cup B) = P(A) + P(B) \)

**Remark**: Make a Venn diagram to help you when you cannot figure out the relationship between events or have trouble to calculate the probability for complex events!

2.4 Counting Technique

- **Multiplication Rule**
  If the sets \( A_1, A_2, \ldots, A_n \) contain, respectively, \( n_1, n_2, \ldots, n_k \) elements, there are \( n_1 \times n_2 \times \cdots \times n_k \) ways of choosing first an element of \( A_1 \), then an element of \( A_2, \ldots, \) and finally an element of \( A_k \).

- **Combination Rule**
  The number of possible choices of \( r \) objects taken at same time from a group of \( N \) distinct objects is given by:
  \[
  \binom{N}{r} = \frac{N(N-1)\cdots(N-r+1)}{r!} = \frac{N!}{r!(N-r)!}
  \]

3 Examples

1. Someone claims to be able to taste the difference among bottled, tap, and canned draft beer of the same brand. A glass of each is poured and given to the subject in an unknown order. The subject is asked to identify the contents of each glass. The number of correct identifications will be recorded.
   - (a) Construct a sample space for this experiment.
   - (b) Identify the event that "not more than one correct identification".

2. Eight pizza delivery persons \( d_1, d_2, \ldots, d_8 \) are working in the campus area. Which one will deliver the next pizza order?
   - (a) Specify the sample space.
   - (b) Consider the three events \( A = \{d_1, d_2, d_5, d_6, d_7\} \), \( B = \{d_2, d_3, d_6, d_7\} \), and \( C = \{d_6, d_8\} \). Drew the Venn diagram and show these events.
   - (c) The probability that each delivery person will be the one to deliver the next pizza order are:
     \[
     P(d_1) = .16 \quad P(d_2) = P(d_3) = P(d_4) = .08
     \]
     \[
     P(d_5) = P(d_6) = P(d_7) = P(d_8) = .15
     \]
   - give the composition and determine the probability of (i) \( BC \); (ii) \( A \text{ or } C \); (iii) \( \overline{A} \text{ or } C \).

3. In a class of 64 seniors and graduate students, 38 are men and 15 are graduate students of whom 8 are women. If a student is randomly selected from this class, what is the probability that the selected student is
   - (a) a senior?
   - (b) a male graduate student?

4. After a preliminary screening, the list of qualified jurors consists of 10 males and 7 females. The 5 jurors the judge selects from this list are all males. Did the selection process seem to discriminate against females? Answer this by computing the probability of having no female members in the jury if the selection is random.