Test II Key

1. (i) The distribution of $X+Y$ is

$$\begin{array}{rcccccc} X+Y & = & 2 & 3 & 4 & 5 & 6 \\ \text{prob} & = & .16 & .44 & .25 & .10 & .05 \end{array}$$

(ii) $P(X+Y=3) = .44$ is largest

(iii) $P(Y=2) = .20 + .15 + .05$ (2nd column) $= .40$

(iv) $P(X=2|Y=2) = \frac{P\left((X,Y)=(2,2)\right)}{P(Y=2)} = \frac{.15}{.40} = \frac{3}{8} = .375$

(v) given $Y=2$, the conditional distribution of X is

$$\begin{array}{rccc} \text{value} = & 1 & 2 & 3 \\ \text{prob.} = & \frac{1}{2} & \frac{3}{8} & \frac{1}{8} \end{array}$$

(divide 2nd column by its column sum).

So $\mathrm{E}(X|Y=2) = 1.\frac{1}{2} + 2.\frac{3}{8} + 3.\frac{1}{8} = \frac{13}{8}$

2.a)

$$\mathrm{E}(3X - Y + 4) = 3\mathrm{E}(X) - \mathrm{E}(Y) + 4 = 15 - 7 + 4 = 12$$

$$\mathrm{Var}(3X - Y + 4) = 9\mathrm{Var}(X) + \mathrm{Var}(Y) + 0 = 27 + 2 = 29$$

$W = 3X - Y + 4$ is normal with $\mu = 12$ and $\sigma = \sqrt{29}$. So

$$P(W > 0) = P\left(\frac{W-12}{\sqrt{29}} > \frac{-12}{\sqrt{29}}\right) = P(Z > -2.23) = P(Z < 2.23) = .9871$$

2.b) (i) significance level α is the distance at $\mu = 10$ from curve up to line $y = 1$. It is .25 or thereabouts in the sketch.

(ii) 2. $\mu < 10$ because $L(\mu)$ gives the probability of accepting H_0 for each μ and you want this to be small for $\mu < 10$.

3.a)

$$\overline{X} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 6.32 \pm 1.734 \frac{.26}{\sqrt{19}} = 6.32 \pm .10 = (6.22, 6.42)$$

Here the degrees of freedom is $\nu = n - 1 = 18$.

3.b) Use the formula

$$E = z_{.05} \frac{\sigma}{\sqrt{n}}$$

with $E = 10,\ \sigma = 60,\ z_{.05} = 1.645$ (from t-table at $\nu = \infty$) and solve for n: $n = 97.42$. Answer: 98.

4.a) Calculate the differences between corresponding lifetimes, $7.1 - 6.8 = 0.3$ etc. You get .3, .4, $-.1$, .7, .1, 0 with mean $\overline{d} = .23$ and sample SD $s_d = .29$. Calculate test statistic

$$T = \frac{\overline{d}}{s_d/\sqrt{n}} = \frac{.23}{.29/\sqrt{6}} = 1.94$$

Compare with $t_\alpha = t_{.05} = 2.015$ ($\nu = 5$). Conclusion: not enough evidence to reject H_0 at the 5% level, so we conclude rust inhibitor A is not significantly better than rust inhibitor B.

4.b) $d = \frac{|\mu - \mu_0|}{\sigma} = \frac{.6}{1.2} = .5$. Use Table 8b) to find $\beta \approx .30$.

5.a) The test uses a Z test statistic with observed value $\frac{11.64-12}{1.3/\sqrt{50}} = -1.96$. Negative values of Z give evidence for the alternative hypothesis. So the P-value is $P(Z < -1.96) = .025 = 2\frac{1}{2}\%$. Reject H_0 at any significance level $> 2\frac{1}{2}\%$ and accept H_0 at any significance level $< 2\frac{1}{2}\%$.

5.b) The test statistic is

$$F = \frac{s^2_{\text{big}}}{s^2_{\text{small}}} = \frac{4.7^2}{3.8^2} = 1.53$$

Compare with $F_{.05}$ at $13 - 1 = 12$ df in numerator and $11 - 1 = 10$ df in denominator. This cutoff point of the F-distribution is 2.91. So accept H_0.