Key to Sample Stat 224 Test I

1. a)

1	*	3	5	6	6
2	*	2	7		
3	*	1	4	5	5

1. b) The answer to both questions is YES.

2. (i) The numbers in the Venn diagram are 17(A only), 16(A B only),11(B only), 8(A C only), 23(A B and C), 10(B C only), 9(C only), and 6(outside A B C).

(ii) $16 + 10 + 8 = 34$.

(iii) $P(B|C) = N(B \cap C)/N(C) = 33/50 = .66$

(iv) $P(B) = 60/100 = .6$ and, from (iii), $P(B|C) = .66$. So C is favorable for B.

3. a) (i) $P(X \text{ is odd}) = P(X = 1) + P(X = 3) = .1 + .3 = .4$

(ii) $E(X) = 1 \times .1 + 2 \times .2 + 3 \times .3 + 4 \times .4 = 3$

(iii) $E(X^2) = 1^2 \times .1 + 2^2 \times .2 + 3^2 \times .3 + 4^2 \times .4 = 10$

(iv) $\text{Var}(X) = E(X^2) - E(X)^2 = 10 - 3^2 = 1$

3. b) (i) #favorable/#total = ${}_{10}P_6/10^6 = .1512$

(ii) This is the complementary event to part (i). So the required probability is

$$1 - .1512 = .8488$$

4. a) 5 accidents per year is equivalent to 5/365 accidents per day and to $30 \times 5/365 = .411$ accidents per month (30 days). So the random variable X = number of accidents in a given month is Poisson with parameter $\lambda = .411$. We want $P(X = 0) = e^{-.411} = .663$ (using the Poisson formula—you can't use the Poisson Table here because it has no entry for $\lambda = .411$).

b) $|X - 12| \geq 8 \iff x \leq 4$ (5 values: 0 thru 4) or $X \geq 20$ (29 values: 20 thru 48). So 34 values of X satisfy $|X - 12| \geq 8$.

X has mean $\mu = np = 12$ and standard deviation $\sigma = \sqrt{np(1-p)} = 3$.

(i) Since 3 is $k\sigma$ with $k = 1$, Chebychev says $P(|X - 12| \geq 3) \leq 1$.

(ii) Since 8 is $k\sigma$ with $k = \frac{8}{3}$, Chebychev says $P(|X - 12| \geq 8) \leq \left(\frac{3}{8}\right)^2 = .141$.

Note that in case (i) Chebychev doesn't tell you anything you didn't already know.

5. a) You would compute the probability of 4 or more failures under the assumption the manufacturer's claim is true. (The smaller this probability is, the less likely you would be to get the observed outcome of 4 failures and the more you would doubt the manufacturer's claim.)

We want $P(X \geq 4)$ for X binomial with parameters $n = 20$ trials (each chip constitutes one trial) and $p = .05$ (a "success" is a chip failure and it has probability 5% assuming the manufacturer's claim). Using the binomial tables,

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - .9841 = .0159$$

b) This is " sampling without replacement" which leads to the hypergeometric distribution.

$$P(\text{exactly 3 spades}) = \binom{13}{3}\binom{39}{2}/\binom{52}{5} = 286 \times 741/2598960 = .0815$$

Note that if it was " sampling with replacement" the number of spades would have a binomial distribution and the answer would be

$$\binom{5}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = .0879$$

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