1. a) 

\[
\begin{array}{ccc}
1 & * & 3 \ 5 \ 6 \ 6 \\
2 & * & 2 \ 7 \\
3 & * & 1 \ 4 \ 5 \ 5 \\
\end{array}
\]

1. b) The answer to both questions is YES.

2. (i) The numbers in the Venn diagram are 17(A only), 16(A B only), 11(B only), 8(A C only), 23(A B and C), 10(B C only), 9(C only), and 6(outside A B C).

(ii) \(16 + 10 + 8 = 34\).

(iii) \(P(B|C) = \frac{N(B \cap C)}{N(C)} = \frac{33}{50} = .66\)

(iv) \(P(B) = \frac{60}{100} = .6\) and, from (iii), \(P(B|C) = .66\). So C is favorable for B.

3. a) (i) \(P(X \text{ is odd}) = P(X = 1) + P(X = 3) = .1 + .3 = .4\)

(ii) \(E(X) = 1 \times .1 + 2 \times .2 + 3 \times .3 + 4 \times .4 = 3\)

(iii) \(E(X^2) = 1^2 \times .1 + 2^2 \times .2 + 3^2 \times .3 + 4^2 \times .4 = 10\)

(iv) \(\text{Var}(X) = E(X^2) - E(X)^2 = 10 - 3^2 = 1\)

3. b) (i) \(#\text{favorable}/#\text{total} = \frac{10 P_0}{10^6} = .1512\)

(ii) This is the complementary event to part (i). So the required probability is 

\[1 - .1512 = .8488\]

4. a) 5 accidents per year is equivalent to 5/365 accidents per day and to \(30 \times \frac{5}{365} = .411\) accidents per month (30 days). So the random variable \(X = \text{number of accidents in a given month}\) is Poisson with parameter \(\lambda = .411\). We want \(P(X = 0) = e^{-\lambda} = .663\) (using the Poisson formula—you can’t use the Poisson Table here because it has no entry for \(\lambda = .411\)).

b) \(|X - 12| \geq 8 \iff x \leq 4\) (5 values: 0 thru 4) or \(X \geq 20\) (29 values: 20 thru 48). So 34 values of \(X\) satisfy \(|X - 12| \geq 8\).
$X$ has mean $\mu = np = 12$ and standard deviation $\sigma = \sqrt{np(1-p)} = 3$.

(i) Since 3 is $k\sigma$ with $k = 1$, Chebychev says $P(|X - 12| \geq 3) \leq 1$.

(ii) Since 8 is $k\sigma$ with $k = \frac{8}{3}$, Chebychev says $P(|X - 12| \geq 8) \leq \left(\frac{3}{8}\right)^2 = .141$.

Note that in case (i) Chebychev doesn’t tell you anything you didn’t already know.

5. a) You would compute the probability of 4 or more failures under the assumption the manufacturer’s claim is true. (The smaller this probability is, the less likely you would be to get the observed outcome of 4 failures and the more you would doubt the manufacturer’s claim.)

We want $P(X \geq 4)$ for $X$ binomial with parameters $n = 20$ trials (each chip constitutes one trial) and $p = .05$ (a “success” is a chip failure and it has probability 5% assuming the manufacturer’s claim). Using the binomial tables,

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - .9841 = .0159$$

b) This is “sampling without replacement” which leads to the hypergeometric distribution.

$$P(\text{exactly 3 spades}) = \binom{13}{3} \binom{39}{2} / \binom{52}{5} = 286 \times 741 / 2598960 = .0815$$

Note that if it was “sampling with replacement” the number of spades would have a binomial distribution and the answer would be

$$\binom{5}{3} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 = .0879$$

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