Sample Test 2 for Stat. 224

1. a) Using the following 10 random digit pairs, simulate 10 values of the discrete random variable X with probability density function

X =	0	1	2	3
prob. =	.12	.12	.60	.16

Random Digits

1 6 8 7 1 6 5 5 4 9 6 0 1 4 1 9 9 2 5 2

How many 2's in your list of simulated values? _____

b) Three independent normal random variables X,Y,Z have respective mean and standard deviation as follows

random variable	mean	standard deviation
X	2.0	0.8
Y	1.2	0.2
Z	1.6	0.4

Find

(i) E(3X - 2Y + Z)

(ii) Var(3X - 2Y + Z)

What distribution does the random variable 3X - 2Y + Z have?

2. a) A pair of continuous random variables X,Y have the joint probability density function

$$f(x,y) = 3/4\ (x+y)$$

on the triangular region with vertices (0,0), (1,0) and (1,2), = 0 elsewhere. (Thus the hypotenuse of the triangle has equation y = 2x.)

Fill in the marginal p.d.f. for X

$$f(x) = \begin{cases} 0 & x < 0 \text{ or } x > 1 \\ \_\_\_\_\_ & 0 < x < 1 \end{cases}$$

b) The probability that a standard normal random variable Z lies between a and -a is .8664.

Find a.

3. a) The manufacturer of a new fiberglass tire claims that they will last longer than the old tires which had an average life of 40,000 miles. To check this claim, a sample of 12 tires is tested with their lifetimes (in 1000's of miles) being as follows

Tire	1	2	3	4	5	6	7	8	9	10	11	12
Life	38.1	40.2	42.8	36.5	42	39.8	42	41	44.8	37.2	43	41

Test the manufacturer's claim at the 5% significance level.

b) The P-value for the hypothesis test $H_0 : \mu = \mu_0$ versus $H_1 : \mu < \mu_0$ turned out to be .042.

(i) Would you reject the null hypothesis at the the 5% significance level?

YES NO (Circle one)

(ii) If the alternative hypothesis was $H_1 : \mu \neq \mu_0$ and the exact same sample data were obtained, what would the

P-value be now? _____

(iii) If you were trying to persuade your boss that the alternative hypothesis was true, would you prefer a

large or small P-value? (Circle one)

4. a) There are two different choices of a catalyst to stimulate a chemical process.To test whether the variance of the yield is the same no matter which catalyst is used, a sample of 10 batches is produced using the first catalyst and 12 using the second. If the resulting data is $S_1^2 = .14$ and $S_2^2 = .28$, can we reject, at the 10% significance level, the hypothesis of equal variance?

b) An ordinary 6-sided die is rolled repeatedly until the first 6 is obtained. This experiment is done 100 times and $\overline{X}$ denotes the average number of rolls needed in the 100 experiments.

Using the Central Limit Theorem, find P($\overline{X}$ > 6.5).

[Hint. For a geometric random variable X with success probability p, E(X) = 1/p and Var(X) = $(1-p)/p^2$.]

5. The local Democratic Party Committee wants to determine the proportion p of the electorate favoring their candidate. If they want 95% confidence that their sample proportion is correct within ±2 percentage points, how large a sample will be needed? _____

Suppose now a random sample of this size is surveyed and it turns out that 56% of the sample favor the Democrats.

Explain briefly in your own words

(i) why it is not technically correct to say that "The probability that p lies between 54% and 58% is .95".

(ii) the precise sense in which it is correct to say that p lies between 54% and 58% with confidence .95.