Nonparametric Methods for Detecting Structure and Dynamics of Earth’s Deep Interior

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Connection with Grace

- Ph.D with Chong Gu (First generation)
- Postdoc with Jun S Liu (Second generation)

Data-driven computational method created

A U.S. statistician has created a data-driven computational approach that's revealing secrets about the inner Earth, as well as gene expression.

Happy Birthday
Introduction: The Seismological Discipline .................................................................................................................. 4

Grand Challenges for Seismology ............................................................................................................................... 10

Grand Challenge 1. How Do Faults Slip? ..................................................................................................................... 11

Grand Challenge 2. How Does the Near-Surface Environment Affect Natural Hazards and Resources? ............ 17

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Sustaining a Healthy Future for Seismology .................................................................................................................. 59
The Mysterious Inner Core

Seismology reveals that Earth’s inner core is surprisingly complex. Although small (about the size of Earth’s moon), the inner core plays an important role: its progressive freezing generates compositional buoyancy by expulsion of light alloy components into the liquid outer core, which serves as an energy source for the outer core convection that maintains Earth’s magnetic field. In the past two decades, seismic analyses have revealed variations in elastic properties of the inner core both radially and laterally, including multiscale variations in attenuation and anisotropy. To first order, the inner core has an overall anisotropic structure, such that waves travel faster and are more attenuated from pole to pole when paralleling the equatorial plane. But, the central region of the inner core has a distinct orientation of anisotropy, and the outermost region is almost isotropic. Large-scale lateral heterogeneities occur both in latitude and longitude in the outer portions of the inner core; seismic velocities are higher...
EARTH’S STRUCTURE
Earth Structure
Keith E. Bullen’s Earth Model

- Bullen (1940, 1942) divided Earth into seven major layers and labeled them as A, B, C, D, E, F, G.

  Sub-layers near major layers D were labeled as D' and D''.

THE PROBLEM OF THE EARTH’S DENSITY VARIATION*

By K. E. Bullen

The seismic wave velocity distributions as given in the new paper of Jeffreys show fairly steady gradients between the base of the earth’s crust and a depth of 0.94 R, where R is the radius of the subcrustal material. It will be convenient to refer to the crustal layers as, compositely, layer A, and the portion of the earth between mean depths of 33 and 413 km. as layer B; the portion between depths of 413 and 984 km. will be referred to as layer C, and that between the last-mentioned depth and the central core as layer D. The layers A, B, C, and D together constitute the earth’s “mantle.” It may be that the 20° discon-
Earthquake Waves

Van der Hilst et al (2007) Science
USArray - Earthscope

Rapid deployment of large number of high-quality seismometers world-wide
Seismic Event Data

Seismic Event Information at the DMC

- About Event Catalogs

Event File Formats
- ISF format with DMC extensions
- WEED Summary File Format

Event Search Tools
- IEB - IRIS Earthquake Browser
- JWEED
- Quake Maps
- Seismicity - Event Query
Exploring the Inner Earth using Seismic Waves

- The limitations of forward modeling
  (1) prior knowledge of earth structure
  (2) signal has sufficient amplitude
  (3) only practical for a small dataset

- Inverse modeling methods that are geophysical sound, mathematical elegant, statistical rigorous are still of rarity.
Isochrons and Image Point Gathers

Van der Hilst et al (2007) Science

Earth
Generalized Radon Transform in Nutshell

- Waveform measurement $u(t, x^s, x^r)$ at time $t$ and with source coordinate $x^s$, receiver coordinate $x^r$
- Travel time function $T(x^s, x^r, y)$ with source coordinate $x^s$, reflected at coordinate $y$, recorded at $x^r$
- Objective: get image at a depth with coordinate $y$
- GRT consists of integrating $u(t, x^s, x^r)$ over a set of curves

$$G(y) = \int \delta(t - T(x^s, x^r, y))u(t, x^s, x^r) dt dx^s dx^r$$
Generalized Radon Transform in Action

• Factors need to be taken into account
  (1) source-receiver geometry:
    slowness vectors
    \[ p^s(y), p^s(x^s), p^r(y), p^r(x^r) \]
    \[ V^s(y) = 1/|p^s(y)|, \alpha^s(y) = p^s(y)/|p^s(y)| \]
    \[ p^m(y) = p^s(y) + p^r(y) \]

(2) properties of Earth’s mantle and crust
  mass density \( \rho(x) \)  stiffness \( c(x) \)
Generalized Radon Transform in Action

- Start with inhomogeneous elastic wave equation of body force
  \[ \frac{1}{\rho(x)c^2(x)} \frac{\partial^2 u(x,t)}{\partial t^2} - \nabla \frac{1}{\rho(x)} \nabla u(x,t) = f(x,t) \]

- The solution is the Green’s function
- Approximations are applied to get computable formulae.
Common Image Point Gathers

\[ u(x^s, x^r, y) = W(x^s, x^r, y) h_p(x^r) \partial u(x^s, x^r, y) \]

\[ 2[\rho(x^r)V^r(x^r)V^r(y)V^s(y)\rho(x^s)V(x^s)]^{1/2}[\det Q(x^s, y)\det Q(y, x^r)]^{1/2} \]

\[ G(y; \theta, \psi) = \int_{E_m} \frac{u(x^s, x^r, y)}{W(x^s, x^r, y)} \frac{|p^m(y)|^3}{|w(x^s, x^r, y)|} dv^m \]
Generalized Radon Transform in Action

\[ s = \text{source} \]; \( r = \text{receiver} \);
\[ y = \text{image point} \]; \( u = \text{data} \)
Common Image Point Gathers

Image points are predefined
Common Image Point Gathers

$u_1$ $u_2$

$r$ $r$

$s$ $s$
Interpretation of Common Image Point Gathers

- Geophysical view: the stack of isochrons.
- Mathematical view: weight average of wave functions.
- Statistical view: the strength of the reflectors.
ScS study: data

1,500 earthquakes
1,200 seismic stations
80,000 ScS (SH)
Simple stacking does not work

(Wang et al., JGR, 2006)
2-D image

(Ma et al., JGR, 2007)
Angle Dilation Depth Harmonic Model

- **The image point gathers at angle** $\theta_i$ **and depth** $x_j$ **is** $G_{ij}$

\[
G_{ij} = g(\alpha_i x_j) + \sum_k a_{ik} \cos(w_{ik} \theta_i + \beta_{ik}) + \epsilon_{ij}
\]

- $a_{ik} \sim N(0, \sigma_i^2)$
- $\epsilon_{ij} \sim N(0, \sigma^2)$

- **Gu & Ma (05) Ann Stat**
(Wang et al., JGR, 2006)
D'' layer

(Ma et al., JGR, 2007)
D” layer
Routine 3-D exploration of lowermost mantle now possible!!
along with calculation of formal uncertainties – (Ma et al. JGR, 2006)

(van der Hilst et al., Science, 2007)
Verification of D”

Consistent with previous studies!

Table 1. D” Discontinuity Detection in Our Study Region

<table>
<thead>
<tr>
<th>Figure 1 Region</th>
<th>Study</th>
<th>P or S</th>
<th>D” Thickness, km</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Lay and Helmberger [1983]</td>
<td>S</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>Kendall and Nangini [1996]</td>
<td>S</td>
<td>290</td>
</tr>
<tr>
<td>3</td>
<td>Kendall and Nangini [1996]</td>
<td>S</td>
<td>…</td>
</tr>
<tr>
<td>4</td>
<td>Garnero and Lay [2003]</td>
<td>S</td>
<td>…</td>
</tr>
<tr>
<td>5</td>
<td>Kendall and Nangini [1996]</td>
<td>S</td>
<td>250</td>
</tr>
<tr>
<td>6</td>
<td>Kendall and Shearer [1994]</td>
<td>S</td>
<td>280</td>
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<tr>
<td>7</td>
<td>Ding and Helmberger [1997]</td>
<td>S</td>
<td>200</td>
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<tr>
<td>8</td>
<td>Ding and Helmberger [1997]</td>
<td>P</td>
<td>…</td>
</tr>
<tr>
<td>9</td>
<td>Zhang and Lay [1984]</td>
<td>S</td>
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<td>10</td>
<td>Reasoner and Revenaugh [1999]</td>
<td>P</td>
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<tr>
<td>11</td>
<td>Lay et al. [2004b]</td>
<td>S</td>
<td>264</td>
</tr>
</tbody>
</table>

(Thomas et al., JGR, 2004)
Compare with geochemical evidence
perovskite -> post-perovskite

Ed Garnero (ASU)
Earthquakes Help Take Deep Earth's Temperature

Earth's Temperature 1,000 Miles Under the Surface Hits 6,650 Degrees Fahrenheit
Scientists find Earth's core is really, really hot

BEIJING, April 2 (Xinhuanet) -- Scientists have stuck a thermometer in the Earth's inner core and found it's really, really hot — about 9,000 degrees Fahrenheit, just 1,000 degrees cooler than the sun's surface.
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EARTH’S DYNAMICS
Earth Structure
The Core

- Core (Oldham 1906)
  - studied S-wave.
  - Inner Core:
    - I. Lehmann (1936): Studied the P-wave. Those detected waves are reflected from the inner core boundary.
    - Birch (1952): The inner core is made up of crystalline iron (depth ≈ 5150-6360 km).
    - **Dziewonski and Gilbert (1971):** Existence confirmed via normal mode analysis.
  - Outer Core:
    - Birch (1952): The outer core is composed of a liquid iron alloy (depth ≈ 2890-5150 km).
    - Today: liquid iron alloy + some light elements.
Rotation of the inner core

Nicotine addiction: The smoking gun
Magnetic superconductors
The Wingless receptor
SUPERROTFATION

It has been observed that differential travel times thought the inner core along certain paths have changed over the past four decades.

- Possible Explanation: 
  - The inner core is not completely homogenous, it possesses regions with distinct seismic velocities.
  - The inner core rotates different than the rest of the Earth (super rotation).

- Goal of the research:
  - to resolve the structure in the inner core
  - and its velocity simultaneously.
Locations of the Alaskan Seismic Network station (black) and the new ARCTIC stations (red) used in this study.
Map of pathways from South Sandwich Islands earthquakes to the Alaska Seismic Network (red) and ARCTIC stations (purple).
Single Station travel time difference
We use a total number of **1165 differential time** measurements covering a time span of **56 years** that are obtained from earthquakes that originate in the **South Sandwich Island** and are recorded at **Alaskan stations**.
Turning point
Model

Travel time difference = Mantle Contribution + Core Contribution

- Mantle Contribution
  - Earthquake epicenter: South Sandwich Islands
  - The seismic network stations: Northern Alaska.
  - The path through the mantle depends mainly on the position of the receiving station and hence denote the mantle contribution can be \( f_{\text{Mantle}}(s_1, s_2) \).

- Core Contribution
  - Northern Alaska
  - The turning point longitude of the ray in the inner core, \( x_2 \).
  - The event time \( t \).
  - The travel time \( a \) of the ray in the inner core
  - The structure we search is a coherent entity and can be described by a moving wave with speed \( v \).
  - The core contribution denoted by \( f_{\text{Core}}(x_2 - v \cdot t, a) \).

- Model: \( y' = f_{\text{Mantle}}(s_1, s_2) + f_{\text{Core}}(x_2 - v \cdot t, a) \)
Both contributions are fitted simultaneously without any constraint:

- Core: $f_1(x_2,t,a)$
- Mantle: $f_2(s_1,s_2)$
STEP 2: SEPARATING THE INNER CORE STRUCTURE FROM ITS TIME EVOLUTION

- The core contribution is constrained to be of the functional form \( \gamma = f_1(x_2-v \cdot t, a) \) representing a coherent object moving with velocity \( v \).

- The velocity, \( v \), is determined via a grid search \( \{v_1, v_2, \ldots, v_N\} \):
  - For each velocity \( v_i \) the core contribution is refitted as \( \gamma = f_1(x_2-v_i \cdot t, a) = f_1(z_i, a) \).
Mantle Contribution

Travel Time Residuals:

Station latitude $s_1$ (°)

Station longitude $s_2$ (°)

- ASN
- ARCTIC
A clear spatial gradient of the inner core with some fine structure is apparent.
Validation
Robustness

- Different partition of observation period:
  - $t_{\text{jump}} = 1975$
  - $t_{\text{jump}} = 1995$

- Different Data Subsets:
  - All data above $t = 1962$
  - All data above $t = 1972$
  - All data above $t = 1990$
  - A data with: $-81.3 < x_2 < -69.8$

- All tests yield:
  - An average velocity $\approx 0.4^\circ$ per year
  - An increase in velocity from about from 0.3/0.4 to 0.5° per year.
Rotation Rate

Average rotation rate of 0.39° per year to the East.

- In agreement with previous studies.
- Both westward and no rotation can be rejected.

Possible change in the rotation rate from 0.24 to 0.56° per year within the last 55 years.

- This yields a minimum angular acceleration $\alpha \approx 1.40 \cdot 10^{-19}$ s$^{-2}$ that has acted on the inner core within the past 36 years due to:
  - The deceleration of the rest of the planet.
  - A real net torque that acts on the inner core.
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