Following Grace from Data Smoothing to Dynamical Systems Identification

Jim Ramsay, McGill University
Some personal background

- A B. Ed. fro U. of Alberta, but with considerable concentration in mathematics and statistics
- Princeton, 1964-1966, Ph. D. in quantitative psychology (multidimensional scaling)
- Statistics in a social science context:
  - Experimental design
  - multivariate statistics
  - Models for testing data, the item characteristic curve
- Functional parameter topics
  - estimated data transformations
  - foundations of measurement (sabbatical leave in Cambridge)
The French Connection

- Sabbatical in Grenoble in 1980-1981
- The l’Lanalyse de Donnee movement (Tukey again)
- Functional analytic approach to multivariate statistics
- And the growing literature on splines
Introduction à l’Analyse des Données

F. CAILLIEZ et J.-P. PAGES
Pierre-Jean Laurent
Université scientifique et médicale de Grenoble

Approximation et optimisation

HERMANN
156, boulevard Saint-Germain, Paris VI
A CORRESPONDENCE BETWEEN BAYESIAN ESTIMATION ON
STOCHASTIC PROCESSES AND SMOOTHING BY SPLINES

BY GEORGE S. KEMELEROE1 AND GRACE WASHILA2
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1. Introduction. Let

\[ L = \sum_{m=0}^{\infty} a_m D^m \]

be a linear differential operator with real constant coefficients and denote the adjoint operator by

\[ L^* = \sum_{m=0}^{\infty} a_m (-D)^m \]

Let \( \{ x_i, i = 1, 2, \cdots, n \} \) be a set of distinct real constants.

Define an \( L \)-spline with knots \( \{ x_i \} \) is a function \( x \in C^{n-1} \) for which

\[ L^* L x = 0 \]

on each open interval \((-\infty, x_i), (x_i, x_{i+1})\); \( x_n = \infty\). Hence an \( L \)-spline consists of piecewise solutions of a 2nd order linear homogeneous differential equation joined at the knots in such a manner as to maintain continuity of all derivatives up to and including the \( (m-2) \)-th. If we were to require that \( x \in C^{n-1} \), then \( x \) would satisfy (1.3) everywhere. Thus, an \( L \)-spline can be looked upon as the "most differentiable" function which satisfies (1.3) on the appropriate open intervals without satisfying it everywhere. Although operators of the form (1.1) are sufficiently general for our present purposes, it should be pointed out that \( L \)-splines are often defined and studied for other linear differential operators, in which case the domain of definition of \( x \) is taken to be a finite closed interval. References [1], [3] and [6] contain extensive bibliographies on splines.

Two common problems for which \( L \)-splines are solutions are the following:

(i) Curve fitting. Given data \( \{ (x_j, y_j), j = 1, 2, \cdots, n \} \) to find a function \( f(t) \) which minimizes

\[ \int_{a}^{b} (L x)^2 \, dt \]

among all functions \( x \) in a certain class for which

\[ x(t_j) = y_j, \quad j = 1, 2, \cdots, n \]

If (1.4) is interpreted as a measure of the roughness of \( x \), then \( x \), if it exists, is the smoothest interpolator to the data.
The Concept (Kimeldorf and Wahba, 1970)

\[
\sum_{j=0}^{n} \sum_{k=0}^{n} [x(t_j) - y_j] b^{jk} [x(t_k) - y_k] + \int_{-\infty}^{\infty} (Lx)^2 \, dt
\]

where

\[
L = \sum j = 0^m a_j (-D^j)
\]
Early work on functional data analysis in France
Getting up to speed at the Institut Fourier
THESE

présentée

A L'UNIVERSITE PAUL SABATIER DE TOULOUSE

pour obtenir

LE GRDE DE DOCTEUR DE SPECIALITE MATHEMATIQUES APPLIQUEES

par

Philippe BESSE

ETUDE DESCRIPTIVE

D'UN PROCESSUS.

APPROXIMATION ET INTERPOLATION

Soutenue le 23 novembre 1979 devant la commission d'examen :

MM. H. CAUSSINUS

M. ATTEIA

J. DAUXOIS

A. POUSSE

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Président

Examinateurs
THÈSE
présentée
A L'UNIVERSITÉ PAUL-SABATIER DE TOULOUSE
pour obtenir
LE GRAD DE DOCTEUR ÉS-SCIENCES (MATHEMATIQUES)
par
Jacques DAUXOIS  Alain POUSSE

LES ANALYSES FACTORIELLES EN CALCUL
DES PROBABILITÉS ET EN STATISTIQUE :
ESSAI D'ÉTUDE SYNTHETIQUE.

Soutenu le 25 mai 1975 devant la Commission d'examen.

MM.  R. HURON  J.R. BARBA  H. CAUSSINUS  P. ETTINGER  J. NEVEU
Président
Examinateurs
cahiers scientifiques  FASCICULE XXVIII
PUBLIÉS SOUS LA DIRECTION DE GASTON JULIA

éléments d'analyse
1
J. DIEUDONNÉ
membre de l'Institut

gauthier-villars
The Green’s Function Connection (Initial value version)

\[ Lx = f \]

implies that

\[ x(t) = x_0(t) + \int_0^t G(w : t)(Lf)(w) \, dw \]

where

\[ Lx_0 = 0 \]

Consequently,

\[ K(s, t) = \int_0^{s \land t} G(w : s)G(w : t) \, dw \]
Some generalizations of the concept

- No need for coefficients $a_j$ to be constant
- No need for the initial value formulation
- No need for $L$ to be linear
- No need for $L$ to be free of unknown parameters
- Sobolev spaces
- The concept of a distribution (Laurent Schwartz)
Early days in functional data analysis

- But we started 40 years out of date: Grenander (1950, 1981)
- Parzen and the transition from time series to functional data analysis
- Representation of functions: smoothing splines all the way
- Human growth, weather and biomechanics
- Ramsay and Dalzell (1991)
- Working with Bernard Silverman
Some Tools for Functional Data Analysis

By J. O. RAMSAY† and C. J. DALZELL

McGill University, Montreal, Canada

[Read before The Royal Statistical Society at a meeting organized by the Research Section on Wednesday, January 16th, 1991, Dr F. Critchley in the Chair]
The Concept Again (Ramsay, Hooker, Campbell, Cao, 2007)

For data in $Y$ and $x(t) = C\phi(t)$

plus differential equation system

$$Dx = f(x)$$

the measure of fit is

$$\|Y - C\Phi\|^2 + \lambda\|CD\Phi - f(C\Phi)\|^2$$

where

$$L = \sum j = 0^m a_j(-D^i)$$
Parameter estimation for differential equations: a generalized smoothing approach

J. O. Ramsay, G. Hooker, D. Campbell and J. Cao

McGill University, Montreal, Canada

[Read before The Royal Statistical Society at a meeting organized by the Research Section Wednesday, May 9th, 2007, Professor T. J. Sweeting in the Chair]
A simple example

- The cranial impact data simulate the effect of a blow to the head arising from, for example, a motorcycle accident.
- Five blows to the skull of a cadaver were administered and the acceleration of the brain tissue recorded.
A simple example

- The shape of the response resembles that of a second order linear system to a point impulse.
- The data were augmented by a forcing function $u$ consisting of short positive pulse.
- The three parameters in vector $\theta$ in the following equation define $L$ and must be estimated from the data, as well as the solution function $x$.
- The basis for representing $x$ contains multiple knots at the beginning and end of the pulse in order to allow for the discontinuity of $u$.
- The coefficients of the expansion are defined as smooth functions $c_k(\theta)$ of the parameters in $\theta$.

$$D^2 x = -\beta_0 x - \beta_1 Dx + \alpha u$$