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BUILDING A MATHEMATICAL MODEL TO PREDICT
TRANSIENT DRILLING TEMPERATURE RESPONSES

by

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ABSTRACT

A transient drilling temperature predicting equation is developed using a statistical model building technique. The functional form of the model is obtained by combining both theoretical considerations and empirical approach. The parameters of this model are estimated and improved by using a sequential procedure for designing the experiments. Confirmatory tests show that the model describes the transient drilling temperature responses remarkably well. The effectiveness of the design procedure for obtaining the "best" estimates of the parameters is also demonstrated.
INTRODUCTION

The objective of this paper is to obtain a drilling temperature predicting equation for transient as well as steady state with speed and feed as the two operating variables. The need for a transient predicting equation is due to the fact that it is too expensive and sometimes infeasible to attain a steady state temperature response especially in the experimental region at high speed and feed (5). A transient temperature predicting equation would be of importance when temperature responses are used to evaluate drill performance.

No prior knowledge of the functional form of the transient predicting equation is available but on the basis of heat transfer mechanism, it is expected that the resulting model will be nonlinear. The functional form of the predicting equation is sought by building a semi-mechanistic model using both theoretical and empirical approach. Having determined the functional form, the parameters of the model are estimated using statistically designed experiments which lead to better estimates. The model and the effectiveness of the designed experiments are checked by confirmatory tests.

1 Underlined numbers in parentheses designate references at the end of the paper.
BUILDING A SEMI-MECHANISTIC MODEL

A predicting equation can be represented mathematically as

\[ E(y) = f(K, \xi) \]  

(1)

where \( E(y) \) is the expected value of the response \( y \)
\( K \) is a vector of parameters
\( \xi \) is a vector of independent variables

and \( f(K, \xi) \) is the response function.

The response \( y \) can be predicted for a given set of independent variables provided that both parameters as well as the functional form of the response function are known. If the exact functional form of the response is unknown then a graduating function, such as polynomials, may be used to approximate the function by a purely empirical approach. Such polynomials may represent the response function adequately within the region of experiments but extrapolation outside this region may not be valid. In the case of a nonlinear response function, instead of using a purely empirical polynomial graduating function it is more appropriate to build a model which is evolved from both empirical as well as theoretical considerations. Such a model can be called a semi-mechanistic model. A semi-mechanistic model will not only represent adequately the response function in the immediate region of the experiments but it also has a better chance of meaningful extrapolation outside the region of experiments than a corresponding empirical model.

A method for building a model is described in Reference (1). This technique works on the principle that the estimated parameters of the system must behave in a prescribed fashion regardless of the settings of the variables. The model building procedure starts by proposing a
simple model based on theoretical knowledge and/or past experience, and specifying the behavior of the parameters. The parameters are estimated by performing a number of designed experiments and the estimates of the parameters are tested for their prescribed behavior. If the behavior of the parameter deviates from the prescribed one then the model is considered to be inadequate and should be modified. This modified model is tested again and the cycle is repeated till an adequate model is obtained. This cycle of building a model is shown in Figure 1.

In this investigation the behavior of the parameters of the preliminary model is assumed to be constant. A factorial design is performed and used to estimate the parameters. These estimates are then treated as responses and a conventional factorial analysis is made to obtain the main effects and interactions. The factorial analysis provides the necessary tests for checking the behavior of the parameters. Since it was specified that the parameters of the model are constant, therefore, the main effects and interactions must be zero. This is true because if the parameters stay constant for any level of the variables, the estimate of parameters obtained from each run of the factorial analysis will be approximately same resulting in zero main effects and interactions. If the estimated parameters do not conform with the prescribed behavior, i.e., stay constant, then the model is inadequate and must be modified.
of the other variables. If the model is found to be inadequate, we will explore for the possible relationships between $K_1$ and $K_2$, and speed and feed.

A $2^2$ factorial experiment was performed choosing 92 rpm and 170 rpm as the two levels for speed and 0.005 ipr and 0.009 ipr for feed. Observations of temperature were taken at five different levels of cutting time, viz. 10, 20, 30, 40 and 50 seconds. The experimental results are shown in Table 1. Each observation in this table is an average of three replicated tests except for the observation at 50 seconds for Run 3 which is an average of only two replicates. This gives a total of 59 observations shown in Appendix A.

The parameters $K_1$ and $K_2$ are estimated by the method of non-linear least squares for each run. The least square estimates of a parameter vector $K$ are obtained by minimizing the error sum of squares, $S(K)$,

$$S(K) = \sum_{u=1}^{n} [ \theta_u - f(K, \xi_u)]^2$$

where $S(K)$ is the sum of squares for $n$ observations

$\theta_u$ is the observed value of temperature at the $u$th trial

and $\xi_u$ are the settings of the independent variables for the $u$th trial.

The minimization of the error sum of squares provides $p$ normal equations which take the following form.

$$\sum_{u=1}^{n} \left[ \theta_u - f(K, \xi_u) \right] \frac{\partial f(K, \xi_u)}{\partial K_i} = 0$$

where $i = 1, 2, 3, \ldots p$ and $p$ denotes the number of parameters in the model. These normal equations are then solved for $K$. 
inadequacy of the proposed model is anticipated because it is well known that drilling temperature depends on speed and feed. However, the factorial analysis provides further useful information that may pinpoint what sort of modifications are necessary for improving the model. In the present case, the factorial analysis clearly indicates that \( K_1 \) depends on speed and feed but not on the interaction. It is obvious that \( K_2 \) also depends on speed and feed but it is not clear whether the interaction affect on \( K_2 \) is significant.

**Modification of Parameters \( K_1 \) and \( K_2 \)**

Since \( K_1 \) depends on speed and feed but not on the interaction, the functional relationship between \( K_1 \) and speed and feed is postulated as follows:

\[
K_1 = b_1 N f^b_2 b_3
\]  \hspace{1cm} (5)

The choice of the exponential form for \( K_1 \) is primarily due to the prior knowledge that this model should be adequate for the steady state cutting temperature. It should be noted that when cutting time is very large, the temperature predicting equation \( \theta = K_1 \left[ 1 - \exp(-K_2 t) \right] \) is equal to \( K_1 \) or \( \theta = K_1 = b_1 N^b_2 f^b_3 \). This equation can be written in a linear form by taking logarithms as

\[
\ln K_1 = \ln b_1 + b_2 \ln N + b_3 \ln f
\]  \hspace{1cm} (6)

therefore \( \ln K_1 \) is used in the factorial analysis shown in Table 3.

The functional relationship between \( K_2 \) and speed and feed accounts for the transient part of the temperature response of which no prior knowledge is available. A simple model which may describe the relationship is

\[
K_2 = b_4 + b_5 N + b_6 f
\]  \hspace{1cm} (7)
if the interaction involving speed and feed is neglected; or

\[ K_2 = b_4 + b_5 N + b_6 f + b_7 Nf \]  \hspace{1cm} \text{(8)}

if the interaction is taken into consideration. Combining equations (5) and (8), a modified drilling temperature equation is

\[ \theta = b_1 N^{b_2 f^{b_3}} \left[ 1 - \exp \left\{ -(b_4 N + b_5 f + b_6 Nf)t \right\} \right] \]  \hspace{1cm} \text{(9)}

An estimation of all the seven parameters of this model showed that the constant \( b_4 \) was extremely small and, therefore, it was decided to redefine the functional relationship for \( K_2 \) by

\[ K_2 = b_4 N + b_5 f + b_6 Nf \]  \hspace{1cm} \text{(10)}

Using this relationship for \( K_2 \), the temperature predicting equation becomes

\[ \theta = b_1 N^{b_2 f^{b_3}} \left[ 1 - \exp \left\{ -(b_4 N + b_5 f + b_6 Nf)t \right\} \right] \]  \hspace{1cm} \text{(11)}

The estimation of parameters of this model was based on all 59 observations shown in Appendix A and the predicting equation is

\[ \theta = 430.54 N^{4.03} f^{4.69} \left[ 1 - \exp \left\{ -\left( 4.62 \times 10^{-4} N + 6.35 f - 0.155 Nf \right) t \right\} \right] \]  \hspace{1cm} \text{(12)}

Checking Adequacy of the Model by "Lack of Fit" Test and "Plots of Residuals"

The adequacy of a model can be tested by comparing the unexplained variation with an estimate of \( \sigma^2 \) using F-statistic. The F-test is not effective for a nonlinear model, but it can be used as an indication of the adequacy of the model. The ANOVA Table for "lack of fit" test is shown in Table 4.

The calculated F-statistic is 2.47 while the tabulated value of F is 2.52 which indicates that the lack of fit test is not significant.

A residual which is defined as the difference between the observed
value and the predicted value can be used to indicate the amount of variation which cannot be explained by the model. The plot of residuals is a good indication of the adequacy of the model both for linear and non-linear models. Two plots of residuals are shown in Figure 3. Figure 3a shows the residuals versus the predicted values of temperature and Figure 3b shows the residuals versus cutting time. Both plots of residuals appear to be random, therefore, there is no reason to doubt the adequacy of the model on the basis of the data.
DESIGNING EXPERIMENTS FOR THE "BEST" PARAMETER ESTIMATION

When parameters of a model are estimated from a set of observations, the location of these experimental points may cause poor estimation of the parameters. To improve the estimation of the parameters it is possible to design the experiments for an adequate model so that the next experiment can be sought to yield a better estimate of the parameters until the "best" parameters are obtained. The "best" parameter estimation is, in a sense, the "best" for a particular model. The criterion used to determine the next experiment for the "best" parameter estimation is the "maximum determinant".

Preliminary data taken from Table 1 are used to start the design procedure. These data constituting a $2^3$ factorial design are shown in Table 5 as eight trials including 24 tests. Based on the model shown in equation (11) and the data for the eight trials, the least square estimates are obtained as follows:

$$
\begin{align*}
b_1 &= 479.87 \\
b_2 &= 0.41347 \\
b_3 &= 0.50516 \\
b_4 &= 5.796 \times 10^{-4} \\
b_5 &= 8.1257 \\
b_6 &= -0.051389
\end{align*}
$$  \hspace{1cm} (13)

Before starting the actual design procedure for the "best" parameter estimation, sum of squares surface and the method for obtaining the joint confidence region and limits are discussed. Sum of squares surface is explored because it is possible that the surface may have multiple minima and the least square solution may converge to a local
instead of the absolute minimum. The joint confidence region and limits are an indication of the uncertainty involved in the estimation of the parameters and will be used to evaluate the estimation situation. The least square estimates of $b$ in equation (13) will be used to illustrate the construction of the sum of squares surface and confidence region and limits.

**Sum of Squares Surface**

Sum of squares, $S(b)$, for a set of observations is defined by equation (3) and can be obtained by calculating the predicted temperature, $\hat{\theta}$, where $\hat{\theta} = f(b, \xi, u)$. A six-dimensional sum of square surface for a six parameter model can be presented in a combination of 15 two-dimensional plots by holding the other four parameters constant. For example, a two-dimensional surface for $b_1$ and $b_3$ is determined by holding $b_2$, $b_4$, $b_5$ and $b_6$ at their least square values and calculating $S(b)$ at different combinations of $b_1$ and $b_3$. Calculation of $S(b)$ for a particular value of $b_1 = 480.0$, $b_3 = 0.7$ is illustrated.

The predicted temperature, $\hat{\theta}$, at the above combination of $b_1$ and $b_3$ is obtained from equation (11) with least square estimates of $b_2$, $b_4$, $b_5$ and $b_6$ shown in equation (13). For example, at the conditions of the first trial, viz. $N = 92$ rpm, $f = 0.005$ ipr and $t = 20$ secs.

$$
\hat{\theta} = f(b, \xi, u) = 480(92)^{0.418}(0.005)^{0.7} \left[ 1 - \exp \left( -\left\{ 5.796(10^{-4})(92) + 8.125(0.005) - 0.0513(92)(0.005) \right\}^{20} \right) \right]
$$

$$
= 58.914^{0}\text{F}.
$$

Since the observed value of temperature is $169.0^{0}\text{F}$, therefore,

$$
\left[ b - \hat{\theta} \right]^2 = (169.0 - 58.914)^2 \text{ for the first trial. Similarly, the value of } \theta \text{ for the other seven trials are calculated and the sum of squares for eight trials is obtained as } S(b) = 2.22(10^5). \text{ A grid of } 10 \times 10 \text{ resulting in 100 different combinations of } b_1 \text{ and } b_3 \text{ was chosen and the}$$

sum of squares at each combination was calculated. The contours of constant sum of squares for \( b_1 \) and \( b_2 \) can then be drawn as shown in Figure 4a. The complete 15 two-dimensional sum of square contours showed only one minimum for each plot with various combinations of parameters. It is, therefore, assumed that there is no possibility of multiple minima in the domain of investigation. Two representative two-dimensional sum of squares contours are shown in Figure 4.

Confidence Region and Limits

The preciseness of the estimates of the parameters is determined from the volume and shape of the joint confidence region. The smaller is the volume of the joint confidence region the better are the estimates.

The approximate joint confidence region is given by

\[ S_c = S_{\text{min}} + s^2 \cdot F_{\alpha} (p, n - p) \]

where \( S_c \) is the value of the critical sum of squares contour

\( S_{\text{min}} \) is the residual sum of squares (or value of sum

of squares with least square estimates)

\( s^2 \) is an estimate of \( \sigma^2 \)

and \( F_{\alpha} (p, n - p) \) is tabulated \((1 - \alpha)100\% \) value of \( F \)-statistic.

To illustrate, the residual sum of squares for the eight trials is 25.6725. This was obtained from equation (3) with least square estimates of \( b \) from equation (13). The estimate of \( \sigma^2 \) is obtained as

\[ s^2 = \frac{S_{\text{min}}}{n - p} = \frac{25.6725}{8 - 6} = 12.83625 \]

and then the magnitude of critical sum of squares contour

\[ S_c = 25.6725 + (12.83625) (6) (19.33) = 1509.67. \]

For a combination of two parameters, say \( b_1 \) and \( b_2 \), a grid of 80 x 80 was laid and sum of squares at all the 6400 combinations was
calculated. The contour of constant $S_c = 1509.67$ was then drawn which is shown in Figure 5a. The region in Figure 5a is long and elongated indicating that estimation situation is poor. Fifteen two-dimensional plots of confidence region were investigated showing the same trend of poor estimation.

Since a complete picture of the joint confidence region for all the six parameters is difficult to obtain, therefore, individual confidence limits are also used to evaluate the estimation situation in the course of this investigation. The individual confidence limits for each parameter is defined as

$$\text{Confidence Limits} = b_i \pm t_{\alpha/2} \sqrt{V(b_i)}$$  \hspace{1cm} (15)

where $b_i$ is a particular parameter

$t_{\alpha/2}$ is a tabulated value of $t$ - statistics

and $V(b_i)$ is the variance of $b_i$ which is equal to the square root of the $i$th diagonal element of $(X_n^T X_n)^{-1}s^2$.

$X_n$ is defined in equation (16).

To illustrate, the 95% confidence limits for $b_3$ for the eight trials are calculated. The $(X_8^T X_8)^{-1}$ matrix was calculated and 3rd diagonal element was $8.97(10^{-5})$. Then

$$\text{Confidence Limits} = b_3 \pm 4.304 \sqrt{8.97(10^{-5})(12.83625)}$$

$$= 0.50516 \pm 0.142$$

95% confidence limits for all the parameters are shown in Figure 6.

**Determinant Criterion**

The determinant criterion works on the principle that the maximization of determinant of $[X_{n+1}^T X_{n+1}]$ leads to a smaller joint confidence region associated with the parameters of the model(2). To
explore the maximization of the determinant of \( [X'_{n+1}X_{n+1}] \) the derivative matrix \( X_n \) is first defined by equation (16).

\[
X_n = \begin{bmatrix}
x_{11} & x_{21} & x_{31} & \cdots & x_{p1} \\
x_{12} & x_{22} & x_{32} & \cdots & x_{p2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_{1n} & x_{2n} & x_{3n} & \cdots & x_{pn}
\end{bmatrix}
\]  

(16)

where

\[
x_{iu} = \frac{\partial f(b, \xi_u)}{\partial b_i} \]

(16a)

and \( i = 1, 2, \ldots, p \) where \( p \) is the number of parameters

and \( u = 1, 2, \ldots, n \) where \( n \) is the number of trials

For the calculations of the maximization of the determinant, a limit on the region of experiments, called the "operability region", was placed in order to have a limited number of calculations. The operability region in this case is chosen within a drill penetration of 1.5". The range of speed is 75, 92, 130, 170, 230, 285 and 400 rpm while available feed levels are .005, .007, .009, .013 and .015 ipr. Cutting time was varied from 10 to 120 seconds in steps of 10 seconds. The operability region is shown in Figure 7.

The maximization of the determinant \( [X'_{n+1}X_{n+1}] \) could be best explained by an example in the determination of the ninth trial.

An example

The first step to search for the ninth trial is to find the derivative
matrix $X_n$ where $n = 8$. Each element in the derivative matrix $X_n$ is obtained by evaluating $x_{iu}$ for the conditions of the $u$th trial and the current least square values of $b$. For instance, $x_{43}$ is calculated by differentiating equation (11) with respect to $b_4$

$$x_{43} = \left. \frac{\partial f(b, \xi_u)}{\partial b_4} \right|_{b}$$

$$= b_2 b_3 \left[ \begin{array}{c} \exp \left( -\left( b_4 N + b_5 f - b_6 N f \right) t \right) \end{array} \right]$$

Substitution of third trial conditions, viz. $N = 92$ rpm, $f = 0.009$ ipr and $t = 20$ seconds, and the least square estimates of $b$ in equation (13), gives

$$x_{43} = 479.87(92)^{418}(0.009)^{505} \left[ (92)(20) \exp \left( -\{5.79(10^{-4})(92) + 8.125(0.009) - 0.0513(92)(0.009)\} 20 \right) \right]$$

$$= 1.00(10^5)$$

By similar procedure for the evaluation of other elements the derivative matrix is

$$X_8 = \begin{bmatrix}
0.3445 & 749.4 & -873.8 & 9.78(10^4) & 5.365 & -496.1 \\
0.5027 & 1242.2 & -1274.9 & 1.40(10^5) & 4.193 & -719.3 \\
0.4996 & 1086.4 & -1126.6 & 1.00(10^5) & 9.898 & -919.0 \\
0.6710 & 1657.1 & -1512.5 & 1.98(10^5) & 10.661 & -1842.1 \\
0.4290 & 933.1 & -1087.9 & 4.75(10^5) & 2.628 & -244.2 \\
0.5772 & 1426.2 & -1463.7 & 4.08(10^5) & 1.241 & -214.9 \\
0.5928 & 1289.3 & -1336.9 & 3.71(10^5) & 3.693 & -346.1 \\
0.7750 & 1914.9 & -1747.7 & 6.06(10^5) & 3.313 & -582.0
\end{bmatrix}$$

The next step is to search for a $X_9$ matrix which is obtained by adding another row to the $X_8$ matrix already known. For example, at $N = 75$ rpm, $f = 0.005$ ipr and $t = 10$ seconds the first element of the ninth row is
\[ x_{19} = \frac{\partial f (b_1, \ldots, b_d)}{\partial b_1} \]

\[ = b_2 b_3 \left[ \frac{1}{1 - e^{-(b_4 N + b_5 f + b_6 N f) t}} \right] \]

\[ = (75) \cdot 418(0.95) \cdot 505 \left[ 1 - \exp \left( -\left\{ 5.796 \cdot 10^{-4} \cdot 75 + 8.125(0.005) \right\} \right) \right] \]

\[ = 0.199 \]

By similar procedure the ninth row of the derivative matrix \( X_9 \) is obtained as follows:

\[ \begin{bmatrix} 0.199 & 415.07 & -506.92 & 7.85 \cdot 10^4 & 525.66 & -395.0 \end{bmatrix} \]

Hence, the matrix \((X'_9 X_9)\) is

\[
(X'_9 X_9) =
\begin{bmatrix}
2.58 & 6.09 \cdot 10^2 & -6.07 \cdot 10^2 & 4.15 \cdot 10^5 & 23.70 & -3.19 \cdot 10^3 \\
6.09 \cdot 10^2 & 1.43 \cdot 10^7 & -1.43 \cdot 10^7 & 9.84 \cdot 10^6 & 5.41 \cdot 10^4 & -7.57 \cdot 10^6 \\
-6.07 \cdot 10^2 & 1.43 \cdot 10^7 & 1.43 \cdot 10^7 & -1.43 \cdot 10^6 & -5.35 \cdot 10^5 & 7.43 \cdot 10^6 \\
4.15 \cdot 10^5 & 5.51 \cdot 10^4 & -5.53 \cdot 10^3 & 5.14 \cdot 10^6 & 3.18 \cdot 10^5 & -1.46 \cdot 10^5 \\
23.70 & -7.57 \cdot 10^2 & 7.43 \cdot 10^6 & 7.06 \cdot 10^5 & 4.06 \cdot 10^4 & 5.72 \cdot 10^2 \\
-3.19 \cdot 10^3 & 7.43 \cdot 10^6 & 4.06 \cdot 10^4 & 5.72 \cdot 10^2 & \end{bmatrix}
\]

The determinant of \([X'_9 X_9]\) for 230 combinations within the operability region was calculated on a computer. The operating conditions which yield the maximum determinant are \( N = 400 \) rpm, \( f = 0.013 \) ipr and \( t = 10 \) seconds. A list of sample values of determinants at different operating conditions is shown in Table 6.

**Parameter Estimation for the Model**

Drilling temperature responses at the conditions of the ninth trial were obtained by experiment and the parameters were reestimated by including the ninth trial. Estimates of parameters from eight and nine trials were then compared. It was found that the confidence intervals
of parameters $b_5$ and $b_6$ were improving after the ninth trial but not that of $b_1$, $b_2$, $b_3$ and $b_4$ as shown in Figure 6. Tenth and eleventh trials were sought by the same procedure as for the ninth. The complete experimental plan and the estimates of the parameters are given in Table 5. Note that the estimates of the parameters for the nth trial include all the trials from 1st up to the nth trial. For example, the estimates of the parameters for ninth trial are based on the experimental results of the first nine trials in Table 5.

The individual confidence limits based on eleven trials for all parameters have decreased considerably (Figure 6). The joint confidence regions were then reexamined and were found to be greatly improved as shown for a representative region in Figure 5b. The individual confidence interval is plotted against the number of trials in Figure 8 to determine the relative importance of each trial. It can be seen that the confidence interval for all parameters almost reaches a constant value after the eleventh trial and that more experiments may not further improve the estimation. Substitution of the estimates, after eleventh trial, in equation (11) yields the final predicting equation for the transient drilling temperature responses.

$$\theta = 423.96N^{385.446} \left[1 - \exp(-\{3.845(10^{-4})N + 6.957t - 0.0171Nt\})\right]$$

(17)

Note that the maximum determinant criterion revealed that all the necessary trials for the further improvement of the experiment are in the high speed range, i.e., 400 rpm which is the maximum limit on the operability region. This information is in agreement with the prior knowledge that high speed is critically important for transient temperature response.
Comparison of Estimates of Parameters Between Eleven Designed Trials and Twenty not Designed Trials

The precision of the estimates depend on the region of the experiment as well as the number of tests. It is possible that a large number of observations may give a less precise estimate than a small number of well planned experiments if the region of experiments is ill-conditioned. This can be illustrated in the present case by comparing the results of the confidence limits from the twenty trials (4 runs each at 5 different times) shown in Table 1 and those from the eleven trials in Table 5.

The individual confidence limits for each parameter based on twenty trials are shown in Figure 9 while the confidence limits based on eleven designed trials are shown in Figure 6. The confidence intervals for each parameter in Figure 9 are drawn to the scale to correspond with those in Figure 6. The individual confidence limits for each parameter based on twenty trials are much bigger than those based on eleven trials. This shows that the estimates obtained from designed experiments are much better even though with less number of trials. The reason for this is that all the twenty trials are in the low speed and feed region, but in order to obtain a good estimate of the parameters for the transient drilling temperature response the experiments should be performed in the high speed and feed region as well as in the low speed and feed region.

CONFIRMATORY TESTS

The adequacy of the drilling temperature model has been shown analytically and the estimates of the parameters were improved using designed experiments. To confirm the adequacy of the model and to demonstrate the effectiveness of designed experiments, eight experiments
covering the drilling conditions which are representative of the entire operability region were further performed.

A comparison of the predicted and observed temperatures clearly demonstrates that the model adequately describes the drilling temperature. Temperature responses for eight cutting conditions ranging from speed of 92 rpm to 400 rpm and feed of 0.005 to 0.013 ipr are plotted in Figures 10a to h. In these figures temperature is plotted for a fixed speed and feed and the corresponding drill penetrations are also marked. The observed temperatures are shown by circles and the predicted temperature, obtained by using equation (17), by a smooth curve. The predicted temperature curves extend for all cutting conditions up to fifty seconds whence the steady state temperature response can be attained.

Some representative values of observed and predicted temperatures are shown in Table 7. The differences between observed and predicted temperature response are mostly within 5% except at the low cutting time. For example, for $t = 4.0$ seconds with $N = 170$ rpm and $f = 0.009$ ipr the percentage difference between the predicted and observed values is 23.28%. The discrepancy may be attributed to the fact that the designed experiments for parameters estimation took time in steps of 10 seconds resulting in poor estimation in the region less than 10 seconds.

The effectiveness of the designed experiments on the parameter estimation leading to a better transient drilling temperature predicting equation can be demonstrated by comparing the observations with the predicted temperature responses. In the low speed region, e.g., at $N = 170$ rpm and $f = 0.005$ ipr, the predicted temperatures agree well with the experimental data as can be seen in the lower part of Figure 11. However, in the high speed and feed region, only the predicted temperature
response from eleven designed experiments, designated as A, fits well with the observation. No predicted temperature responses agree well with the observed temperature data using non-designed trials.

The largest discrepancy between the predicted temperature responses and the observed data, were shown by response B which was obtained using four low speed and feed combinations at \( t = 20 \) and 40 seconds, and which was the starting \( 2^3 \) factorial design for the best parameter estimation. Predicted temperature responses shown in C, D, E and F were obtained for 3, 12, 13 and 20 trials using four low speed and feed combinations corresponding to a time of 10 and 20 seconds, 10, 20 and 30 seconds, 10, 20, 30 and 40 seconds, and 10, 20, 30, 40 and 50 seconds respectively. The percentage difference between the predicted and the observed values for various combinations of drilling conditions ranging from A to F are shown in Table 8. Note that temperature response F shows good agreement with the experimental data only at low cutting time but starts deviating as time increases. All other responses, i.e., B, C, D and E show much higher deviations.

CONCLUSIONS

1. A transient drilling temperature predicting equation in six parameters with speed and feed as variables has been obtained for a specific drill and workpiece combination. It is expected that the functional form of the equation will be valid for a different drill and workpiece combination except for the values of the parameters. The equation can be used to predict the transient as well as the steady state temperature responses. The required time to reach the steady state value can also be estimated.
2. A statistical model building technique is introduced to build a semi-mechanistic model for the transient drilling temperature equation. Factorial design and analysis is employed to initiate a preliminary model and to pinpoint necessary modifications. Nonlinear least square method is used to estimate the parameters. Adequacy of the model is checked by Analysis of Variance and the plotting of residuals.

3. The "best" parameter estimation is achieved by using statistically designed experiments. Maximum determinant criterion is used to seek the next experiment for better estimates. Sum of square surfaces and joint confidence region are explored to assure the absolute minimum in the region of interest and the improvement of the estimate of the parameters.

4. Eight confirmatory tests covering a range of practical importance were performed to check the adequacy of the model. The experimental data agree remarkably well with the model. A comparison is made between the observations and the predicted temperature responses based on both the designed experiment and the non-designed experiment. The effectiveness of the designed experiment in improving the parameter estimation is clearly demonstrated.
## APPENDIX A

### Experimental Data

<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>Feed (ipr)</th>
<th>Time (secs)</th>
<th>Temperature Replicate, °F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>92.0</td>
<td>0.005</td>
<td>10</td>
<td>110.5</td>
</tr>
<tr>
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Marquardt's Compromise

Sample calculations for obtaining the least square estimates of $K_1$ and $K_2$ using Marquardt's method, based on the data for first run in Table 1, are shown. These calculations were performed by using a computer program "GAUSHAUS". The use of Marquardt's compromise is illustrated by working out the calculations for second iteration.

Before the calculations are started, some initial estimates of the parameters are needed because each element of the derivative matrix is calculated at current values of $K$ (equation 16a). Preliminary estimates were obtained by solving the following two equations which satisfy equation (2).

At $t = 40$,
\[ 204.3 = K^{(0)}_1 \left[ 1 - e^{-K^{(0)}_2 (40)} \right] \]

At $t = 50$,
\[ 214.3 = K^{(0)}_1 \left[ 1 - e^{-K^{(0)}_2 (50)} \right] \]

where the superscript denotes the number of iteration.

An approximate trial and error solution of these two equations gives
\[ K^{(0)}_1 = 236.2 \]
\[ K^{(0)}_2 = 0.05 \]

and \[ \text{Sum of Squares} = 691.94 \]

A parameter $\lambda$ is defined and arbitrarily taken as 0.01. After the first iteration following values were obtained.
\[ k_1^{(1)} = 229.4 \]
\[ k_2^{(1)} = 0.0558 \]

and Sum of Squares

\[ s(k) = 363.7 \]
\[ \lambda = 0.001 \]

The second iteration is started from the final values of the first iteration. Detailed calculations are explained in nine steps.

Step 1 Set up a derivative matrix, \( Z \). Each element of this matrix is calculated from

\[ z_{iu} = \left. \frac{\delta (K, \xi_u)}{\delta K_i} \right|_{K = K^{(1)}} \]

where \( K^{(1)} \) is the value of vector \( K \) at the end of first iteration. For example

\[ z_{24} = \left. \frac{\delta (K, \xi_4)}{\delta K_2} \right|_{K = K^{(1)}} \]

\[ = k_1^{(1)} t \left[ (1 - e^{-k_2^{(1)} t}) \right] \]
\[ = 229.4(40) \left[ 1 - e^{-0.0558(40)} \right] \]
\[ = 984.70950 \]

The \( Z \) matrix is thus obtained as

\[
Z = \begin{bmatrix}
0.42765 & 1312.97700 \\
0.67241 & 1502.97160 \\
0.81250 & 1290.34470 \\
0.89269 & 984.70950 \\
0.93858 & 704.50136
\end{bmatrix}
\]
Step 2 Obtain $Z'Z$ and scale factors

$$B = Z'Z = \begin{bmatrix} 2.9730000 & 4160.78580 \\ 4160.7858 & 7113796.50 \end{bmatrix}$$

and the scale factors are

$$C_{11} = \frac{1}{\sqrt{2.973}}$$

$$C_{22} = \frac{1}{\sqrt{7113796.5}}$$

Step 3 Get the scaled matrix $A$. Each element of $A$ is calculated as

$$a_{ij} = \frac{B_{ij}}{\sqrt{C_{ii} C_{jj}}}$$

$$A = \begin{bmatrix} 1.00000 & 0.90475 \\ 0.90475 & 1.00000 \end{bmatrix}$$

Step 4 Calculate a matrix $Y - h$

where $Y$ is the (nx1) matrix of the observed values of temperature viz. the first run in Table 1, and $n = 5$

and $h$ is the (nx1) matrix of the calculated temperatures

As an example $Y(3,1) = 197.3$ (from Table 1)

while

$$h(3,1) = K_1^{(1)}(1 - e^{-K_2^{(1)} t})$$

$$= 229.4 [1 - e^{-0.0558(30)}]$$

$$= 186.38853$$
Similarly all the other elements of \( h \) were calculated. Subtracting \( h \) from \( y \),

\[
y - h = \begin{bmatrix} 5.06769 \\ 14.74857 \\ 10.95147 \\ -0.44228 \\ -0.96998 \end{bmatrix}
\]

**Step 5** Obtain \( z' (y-h) \) and a scaled matrix \( q \)

\[
z' (y-h) = \begin{bmatrix} 19.677210 \\ 41832.744 \end{bmatrix}
\]

and \( q \) is obtained by dividing each element by scale factors

\[
q = \begin{bmatrix} 11.41211 \\ 15.68432 \end{bmatrix}
\]

**Step 6** Set up an equation

\[
(A + \lambda I) \delta^* = q
\]

and solve for \( \delta^* \). Since \( \lambda = 0.001 \)

\[
\begin{bmatrix} 1.00100 & 0.90475 \\ 0.90475 & 1.00100 \end{bmatrix} \delta^* = \begin{bmatrix} 11.41211 \\ 15.68432 \end{bmatrix}
\]

and

\[
\delta^* = \begin{bmatrix} -15.08329 \\ 29.30156 \end{bmatrix}
\]

Rescale \( \delta^* \) using scale factors \( C_{11} \) and \( C_{22} \)

\[
\delta^* = \begin{bmatrix} -8.74779 \\ 0.01099 \end{bmatrix}
\]
Step 7  The new estimates of $K$ are

\[ K_1^{(2)} = K_1^{(1)} + \delta(1,1) = 220.65221 \]

\[ K_2^{(2)} = K_2^{(1)} + \delta(2,1) = 0.06679 \]

Step 8  Calculate $K_1^{(2)}$ and $K_2^{(2)}$ with $\lambda = 0.0001$ instead of $\lambda = 0.001$ repeating the Steps 6 and 7.

\[ K_1^{(2)} = 220.53268 \]

\[ K_2^{(2)} = 0.06687 \]

Step 9  Calculate

Sum of Square (for $\lambda = 0.001$) = 100.42415

Sum of Square (for $\lambda = 0.0001$) = 104.06450

Using the tests outlined in Reference (6) the value of $\lambda$ for 3rd iterations

\[ \lambda = 0.001 \]

while $K_1^{(2)} = 220.65221$

\[ K_2^{(2)} = 0.06679 \]

and $\quad$ Sum of Squares = 104.7

The third iteration is started with the above values. In all, four iterations were needed when the calculations were stopped because the relative change in sum of squares was less than $10^{-5}$.

The final values are

\[ K_1^{(4)} = 223.18 \]

\[ K_2^{(4)} = 0.066858 \]

\[ \text{Sum of Squares} = 81.82 \]
REFERENCES


8. "GAUSHAUS" University of Wisconsin Computing Center.
LIST OF TABLES

Table 1  Experimental Temperature Responses, °F, on a $2^2$ Factorial Design
Table 2  Least Square Estimates of the Parameters
Table 3  Factorial Analysis on ln$K_1$ and $K_2$
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Table 5  Experimental Plan and Estimates of the Parameters
Table 6  Sample Values of Determinant $[X'_9 X_9]$
Table 7  Comparison of Observed and Predicted Temperatures
Table 8  Percentage Difference Between the Predicted and the Observed Temperatures for Various Combinations of Drilling Conditions
Table 1

Experimental Temperature Responses, °F, on a $2^2$ Factorial Design

<table>
<thead>
<tr>
<th>Run</th>
<th>Speed, N</th>
<th>Feed, f*</th>
<th>Time (second)</th>
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</thead>
<tbody>
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<td></td>
<td>rpm</td>
<td>ipr</td>
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<td>-</td>
<td>-</td>
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<td>181.7</td>
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<tr>
<td>3</td>
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<td>+</td>
<td>175.0</td>
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<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>263.7</td>
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*High Level + 170.0 0.009
Low Level - 92.0 0.005
Table 2

Least Square Estimates of the Parameters

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<th>Speed</th>
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<th>Parameter</th>
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<th>K₂</th>
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Table 4

ANOVA Table for "Lack of Fit" Test

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<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Squares</th>
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<td>Pure Error (by replications)</td>
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<tr>
<td>Lack of Fit (by subtraction)</td>
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<td>14</td>
<td>203.510</td>
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<td>Trials</td>
<td>Speed (rpm)</td>
<td>Feed (ipr)</td>
<td>Time (secs)</td>
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Table 6
Sample Values of Determinant \( \begin{bmatrix} X_9'X_9 \end{bmatrix} \)

<table>
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<tr>
<th>Speed (rpm)</th>
<th>Feed (ipr)</th>
<th>Time (secs.)</th>
<th>Penetration (inches)</th>
<th>Determinant</th>
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<td>186.0</td>
<td>163.4</td>
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<td>283.3</td>
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<td>371.3</td>
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<td>435.9</td>
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<td>483.3</td>
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<td></td>
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<td>504.0</td>
<td>518.0</td>
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Table 8

Percentage Difference Between the Predicted and the Observed Temperature for Various Combinations of Drilling Conditions

Speed = 400 rpm, Feed = 0.013 ipr

<table>
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<tr>
<th>Time (Seconds)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
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<td>-6.70</td>
<td>53.70</td>
<td>33.30</td>
<td>26.80</td>
<td>19.30</td>
<td>5.91</td>
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<tr>
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<td>-8.41</td>
<td>48.20</td>
<td>25.50</td>
<td>21.70</td>
<td>17.10</td>
<td>7.44</td>
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<tr>
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<td>-4.87</td>
<td>42.30</td>
<td>20.00</td>
<td>17.40</td>
<td>15.40</td>
<td>8.90</td>
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<td>1.13</td>
<td>35.70</td>
<td>16.80</td>
<td>15.20</td>
<td>14.80</td>
<td>11.20</td>
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<tr>
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<td>11.10</td>
<td>10.30</td>
<td>11.10</td>
<td>9.70</td>
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<tr>
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<td>2.70</td>
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<td>9.90</td>
<td>9.30</td>
<td>7.52</td>
<td>11.30</td>
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</tbody>
</table>

*Eleven designed trials (from Table 5)

B Eight trials (first eight trials from Table 5)

C Eight trials (four runs each at t = 10 and 20 seconds from Table 1)

D Twelve trials (four runs each at t = 10, 20 and 30 seconds from Table 1)

E Sixteen trials (four runs each at t = 10, 20, 30 and 40 seconds from Table 1)

F Twenty trials (four runs each at t = 10, 20, 30, 40 and 50 seconds from Table 1)
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+ denotes high level

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Number of Trials = 8

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N = 130 rpm  
\( f = 0.007 \) ipr

N = 230 rpm  
\( f = 0.007 \) ipr

N = 400 rpm  
\( f = 0.007 \) ipr

N = 400 rpm  
\( f = 0.013 \) ipr

10g

10h

10e

10f
Figure 11. Comparison of Observed Temperature with the Predicted Temperature Responses Based on Different Sets of Trials