STAT 771 **Quiz 3** **Newton: 4–20–09.**

Record all answers in the blue books. Show your work.

1. Suppose that $Z_0, Z_1, Z_2, \ldots, Z_n$ form a two-state Markov chain on states $\{1, 2\}$, according to a transition probabilities $\{p_{j,k}\}$, and the event $Z_0 = 1$ is known to have occurred. Observables $Y_1, Y_2, \ldots, Y_n$ are conditionally independent of each other given the Markov chain, with Y_i having conditional density $f_k(y)$ given $Z_i = k$. Let $u_n(k) = 1$ for $k \in \{1, 2\}$, and for any $i \in \{1, 2, \ldots, n-1\}$ define the density $u_i(k) = P(y_{i+1}, y_{i+2}, \ldots, y_n | Z_i = k)$ for realizations of the observables. Derive a recursive algorithm to evaluate $u_i(k)$ for all i and all k. Justify steps taken.

2. Proportions y_i/m_i in the table below are realizations of Binomial proportions from a system in which the true underlying probabilities are non-decreasing. Implement the pool adjacent violators algorithm to obtain the maximum likelihood estimate of the underlying probabilities.

i	y_i	m_i
1	2	4
2	1	4
3	2	4
4	3	4
5	4	4
6	2	4

3. Consider a rejection sampling algorithm to generate standard normal variables using Laplace-distributed variables. I.e., the target density is $f(x) = \frac{1}{\sqrt{2\pi}} \exp\{-x^2/2\}$ and the easily sampled density is $g(x) = \frac{1}{2} \exp\{-|x|\}$. Recall that if S is a random sign (taking values $+1$ and -1 with equal probability) and if U is a uniform$(0, 1)$ variable, then $X = S \log(1/U)$ has density g. Show that $f(x) \leq cg(x)$ for some $c \geq 1$, and identify the smallest possible value of c. Describe the rejection sampling algorithm for realizing a standard normal.

4. We have four discrete random variables X, Y, A, and B. The latter two each take K possible values $\{1, 2, \ldots, K\}$. For each possibility x, y of the first two, the computation required to compute $p(x, y) = \sum_{a=1}^{K} \sum_{b=1}^{K} p(x, y, a, b)$ involves about K^2 individual summation operations in the absense of modeling constraints. One model considered involves conditional independencies such that $p(x, y, a, b) = p(x|a)p(y|a)p(a|b)p(b)$ for all arguments. Indicate how this model reduces the computational burden in evaluating $p(x, y)$.