STAT 771 **Quiz 2** **Newton: 3–11–09.**

Record all answers in the blue books. Show your work.

1. Consider a generic missing data problem in which data X and missing data Z have a joint probability density function, $p(x, z|\theta)$, determined by a parameter θ. Let

$$Q_m(\theta) = E\left\{\log p(x, Z|\theta) \,|\, X = x, \theta_m\right\}$$

be as constructed in the E step of the EM algorithm, at iteration m when the current parameter value is θ_m. Let $H_m(\theta) = l(\theta) - Q_m(\theta)$, where $l(\theta) = \log p(x|\theta)$ is the ordinary log likelihood. Taking $\Delta_m(\theta) = H_m(\theta) - H_m(\theta_m)$, argue that $\Delta_m(\theta) \geq 0$ for all θ, and therefore that $l(\theta_{m+1}) \geq l(\theta_m)$ upon taking an M step to position θ_{m+1}. [Do not be overly concerned with regularity conditions on the densities that would allow various manipulations to be valid, but do state the rationale for each step taken.]

2. The following model is a simplification of one arising in the study of how neurotransmitters mediate their effects. We have a collection of mutually independent random variables: for each $i = 1, 2, \ldots, n$, we have two Poisson-distributed variables U_i, V_i with common rate $\lambda > 0$, and we also have a Bernoulli trial W_i having success probability α. The observable count $X_i = U_i + V_i W_i$. Set up an EM algorithm to compute the maximum likelihood estimate of $\theta = (\lambda, \alpha)$ on the basis of realized values $x_1, \ldots, x_n$. [Recall the mass function of a Poisson-distributed variable U is $e^{-\lambda}\lambda^u/u!$ for $u = 0, 1, 2, \ldots$.] Also, find an easily computed method of moments estimate that may be used to initiate the algorithm.