STAT 771 **Quiz 1** **Newton: 2–16–09.**

Record all answers in the blue books. Show your work.

1. The normal linear regression model relates the $n \times p$ design matrix X to the $n \times 1$ response vector Y according to $Y = X\beta + \epsilon$, where ϵ is an $n \times 1$ vector of mean zero, constant variance, uncorrelated normal errors, and β is an unknown $p \times 1$ parameter vector. We know that the least squares estimator $\hat{\beta}$ (also the MLE in the normal model) satisfies the normal equations

$$X^T X \hat{\beta} = X^T Y.$$

   Describe one way that a matrix decomposition can be used to solve for $\hat{\beta}$ without explicitly inverting $(X^T X)$.

2. Recall the zero-inflated-Poisson (ZIP) model, in which the observable count variable $X_i = B_i Z_i$, where B_i is a Bernoulli(1/2) variable and Z_i is an independent Poisson variable with mean θ. Recall also that upon observing a random sample of n ZIP variables, the log likelihood $l(\theta)$ equals

$$l(\theta) = c + n_0 \log\left[1 + \exp(-\theta)\right] - (n - n_0)\theta + t \log(\theta)$$

   where n_0 is the number of zeros, t is the total observed count, and c does not depend on θ.

   (a) Derive the score function and the observed information function.

   (b) Describe the Newton-Raphson algorithm for computing the MLE $\hat{\theta}$.

   (c) Derive the Fisher information.

3. Three measurements X_1, X_2, and X_3 are modeled to be independent and identically distributed draws from Laplace distribution with unit scale and unknown center θ. That is, their common probability density function is

$$f(x|\theta) = \frac{1}{2} \exp\{-|x - \theta|\}$$

   for real x. (Of course, $|u|$ is the absolute value of u.) Evaluate and sketch the log likelihood function $l(\theta)$ upon observing $(x_1, x_2, x_3) = (6.0, 4.0, 7.0)$. What is the maximum likelihood estimate of θ?

4. Let A denote a $p \times p$ symmetric positive definite matrix. The $p-$vector v is called an eigenvector of A if for some real value λ, $Av = \lambda v$. Show that $\lambda > 0$.