1. Sketch the empirical distribution function of the following measurements: \{2.1, 1.2, 6.9, 7.1, 3.6\}. These numbers are the realization of a random sample from a fixed population. Describe how you would compute a 95% bootstrap percentile confidence interval for the mean of this underlying population.

2. A positive random variable \(X\) has probability density

\[f(x) = \frac{2}{\theta} \left(1 - \frac{x}{\theta}\right)\]

for \(x \in (0, \theta)\), and for an unknown parameter \(\theta > 0\). Having observed \(x\), determine and sketch the log-likelihood for \(\theta\) and the score function for \(\theta\). Derive the maximum likelihood estimator. Treating the MLE as a random variable, compute its expected value.

3. Two independent counts \(X\) and \(Y\) will be observed. \(X\) is Poisson distributed with mean \(\alpha \beta > 0\), and \(Y\) is Binomially distributed on a known number of trials \(n\) and an unknown success probability \(\alpha \in (0, 1)\). Having observed \(x\) and \(y\), determine the maximum likelihood estimates of \(\alpha\) and \(\beta\).

4. A sequence of Bernoulli trials \(X_0, X_1, X_2, \ldots, X_n\) is modeled as a two-state Markov chain. That is, for \(i = 1, 2, \ldots, n\),

\[
\begin{align*}
P(X_i = 1 | X_{i-1} = 0) &= \alpha \\
P(X_i = 0 | X_{i-1} = 0) &= 1 - \alpha \\
P(X_i = 0 | X_{i-1} = 1) &= \beta \\
P(X_i = 1 | X_{i-1} = 1) &= 1 - \beta
\end{align*}
\]

for unknown parameters \(\alpha\) and \(\beta\). Treating \(X_0 = 0\) as fixed, derive a formula for the likelihood

\[
L(\alpha, \beta) = P(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n | X_0 = 0, \alpha, \beta) = \prod_{i=1}^{n} P(X_i = x_i | X_{i-1} = x_{i-1}, \alpha, \beta)
\]

Find the MLE’s of \(\alpha\) and \(\beta\).