

Bret Larget

Departments of Botany and of Statistics  
University of Wisconsin—Madison

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# Linear Transformations

- Linear transformations *do not change the quality of a fit.*
- Linear transformations *can make parameters easier to interpret.*
- `birds` are the birds from the birds and bats example.

```
> bats = read.table("bats.txt", header = T)
> birds = with(bats, bats[type == "bird", ])
> birds0.lm = lm(energy ~ mass, data = birds)
> birds.mean = with(birds, mean(mass))
> birds.mean

[1] 263.175

> birds1.lm = lm(energy ~ I(mass - birds.mean), data = birds)
```

# Compare Output

- The R-Squared and residual sd are the same for both fits.
- The intercept is different.

```
> display(birds0.lm, digits = 5)
```

```
lm(formula = energy ~ mass, data = birds)
      coef.est coef.se
(Intercept)  3.31674  3.29882
mass         0.06777  0.01074
---
n = 12, k = 2
residual sd = 5.88610, R-Squared = 0.80
```

```
> display(birds1.lm, digits = 5)
```

```
lm(formula = energy ~ I(mass - birds.mean), data = birds)
      coef.est coef.se
(Intercept)    21.15333  1.69917
I(mass - birds.mean) 0.06777  0.01074
---
n = 12, k = 2
residual sd = 5.88610, R-Squared = 0.80
```

# Predictions

- Use the `predict()` function to make predictions.
- The predicted values are the same.

```
> newbirds = data.frame(mass = c(200, 250, 300))  
> predict(birds0.lm, newbirds)
```

```
      1      2      3  
16.87167 20.26040 23.64913
```

```
> predict(birds1.lm, newbirds)
```

```
      1      2      3  
16.87167 20.26040 23.64913
```

- Transformations by centering are especially helpful for coefficient interpretation when there are interactions in the model.

```
> mass.mean = with(bats, mean(mass))
> mass.mean

[1] 262.675

> bats1.lm = lm(energy ~ I(mass - mass.mean) * type, data = bats)
> display(bats1.lm, digits = 4)

lm(formula = energy ~ I(mass - mass.mean) * type, data = bats)
              coef.est coef.se
(Intercept)      21.1194   1.4552
I(mass - mass.mean)  0.0678   0.0092
typeeBat          2.9198  16.1746
typenBat           0.6269   3.9758
I(mass - mass.mean):typeeBat  0.0219   0.0687
I(mass - mass.mean):typenBat -0.0277   0.0149
---
n = 20, k = 6
residual sd = 5.0408, R-Squared = 0.90
```

## Short Quiz

```
> coef(bats1.lm)
```

```
                (Intercept)                I(mass - mass.mean)
21.11944601                        0.06777464
      typeeBat                                typenBat
      2.91984047                        0.62693918
I(mass - mass.mean):typeeBat I(mass - mass.mean):typenBat
      0.02186199                        -0.02772895
```

- 1 Interpret each intercept coefficient.
- 2 Interpret each slope coefficient.
- 3 Predict the energy for a 20 g echo-locating bat.
- 4 Predict the energy for a 220 g bird.
- 5 Predict the energy for a 420 g non-echo-locating bat.

# Log Transformations

- Log transformations:
  - ▶ *do change* the goodness of fit;
  - ▶ can make a linear model fit better.
  - ▶ require variables with positive numerical values;
  - ▶ are especially useful large values are more variable than small values
- The function `log()` takes the *natural* logarithm.
- Use the `exp()` function to go back to the original units.
- Since  $\exp(1 + x) \approx x$  for  $x$  near 0, interpretations of coefficients after a log transformation of the outcome variable are easier to interpret.

# Birds and Bats Example

```
> lmass.mean = with(bats, mean(log(mass)))
> bats2.lm = lm(log(energy) ~ I(log(mass) - lmass.mean) + type,
+ data = bats)
> display(bats2.lm, digits = 4)
```

```
lm(formula = log(energy) ~ I(log(mass) - lmass.mean) + type,
    data = bats)
```

	coef.est	coef.se
(Intercept)	2.5074	0.0558
I(log(mass) - lmass.mean)	0.8150	0.0445
typeeBat	-0.0236	0.1576
typenBat	-0.1023	0.1142

---

n = 20, k = 4

residual sd = 0.1860, R-Squared = 0.98

- In a comparison of an echo-locating bat to a bird of equal mass, we expect the energy use to be about 2% less.
- In a comparison of a non-echo-locating bat to a bird of equal mass, we expect the energy use to be about 10% less.

# Predictions

```
> newData = data.frame(mass = c(20, 20, 400, 400), type = factor(c("bird",  
+ "eBat", "bird", "nBat")))  
> exp(predict(bats2.lm, newData))
```

1	2	3	4
2.630822	2.569466	30.225643	27.287500

## General Principles:

- 1 Include all input variables expected to be important on scientific grounds.
- 2 Consider including interactions for inputs of large effect.
- 3 It is okay to use judgment in deciding whether or not to include variables.
- 4 (We will revisit more formal methods of variable selection later.)

If time permits, do live in R.

- 654 kids with ages ranging from 3 to 19
- Age is measured in years.
- FEV is forced expiratory volume, a measure of lung capacity, and is measured in liters.
- Height is measured in inches.
- Sex has two levels (female, male)
- Smoking has two levels (nonsmoker, smoker)