

Multiple Linear Regression Case Study

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- Birds and bats must expend considerable energy to fly.
- Some bats use echolocation in flight which also requires energy.
- Other bats eat fruit and do not have the ability to echolocate.
- Scientists studied energy use of several species of birds and bats to examine the relationship between mass and energy expenditure during flight to see if echolocating bats had a higher cost.
- Variables are mass (grams), type (factor with levels bird, eBat, and nBat, latter two for echolocating and non-echolocating), and the response energy (Watts).

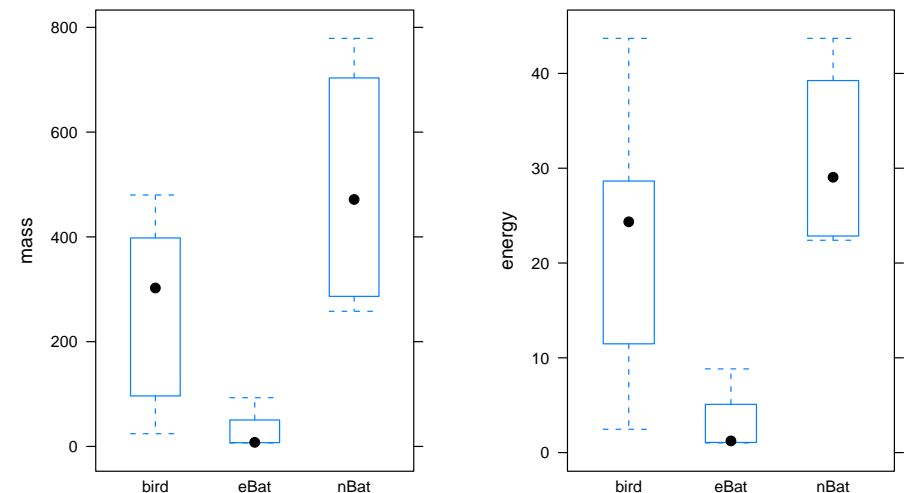
Data

```
> bats = read.table("bats.txt", header = T)
> bats
```

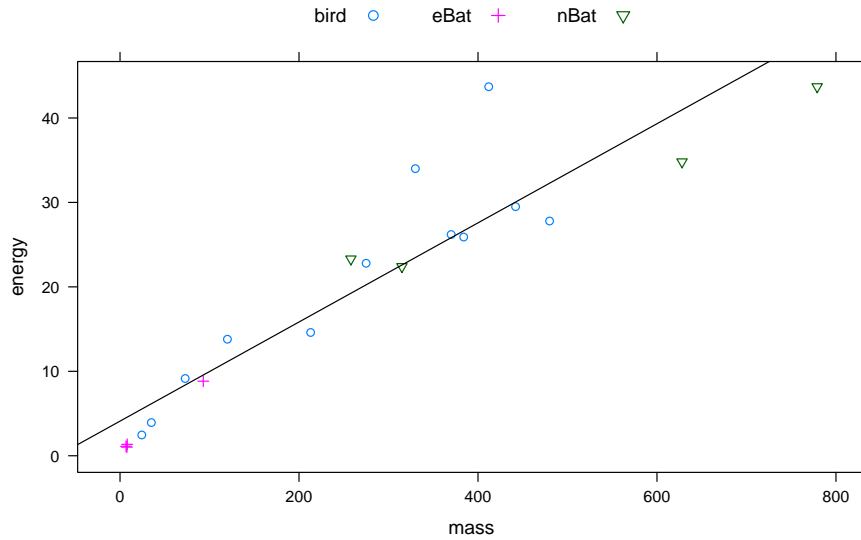
	species	mass	type	energy
1	PteropusGouldi	779.0	nBat	43.70
2	PteropusPoliocephalus	628.0	nBat	34.80
3	HypsignathusMonstrosus	258.0	nBat	23.30
4	EidolonHelvum	315.0	nBat	22.40
5	MeliphagaVirescens	24.3	bird	2.46
6	MelipsittacusUndulatus	35.0	bird	3.93
7	SturmisVulgaris	72.8	bird	9.15
8	FalcoSpaverius	120.0	bird	13.80
9	FalcoTinnunculus	213.0	bird	14.60
10	CorvusOssifragus	275.0	bird	22.80
11	LarusAtricilla	370.0	bird	26.20
12	ColumbaLivia	384.0	bird	25.90
13	ColumbaLivia	442.0	bird	29.50
14	ColumbaLivia	412.0	bird	43.70
15	ColumbaLivia	330.0	bird	34.00
16	CorvusCrytoleucos	480.0	bird	27.80
17	PhyllostomasHastatus	93.0	eBat	8.83
18	PlecotusAuritus	8.0	eBat	1.35
19	PipistrellusPipistrellus	6.7	eBat	1.12
20	PlecotusAuritus	7.7	eBat	1.02

- Notice that both mass and energy span different orders of magnitude.
- The two bat types are quite different in mass.
- Birds fill the gap.
- Each observation corresponds to a single study.
- Some studies are on the same species.

Box-and-Whisker Plots



Scatterplot



Observations

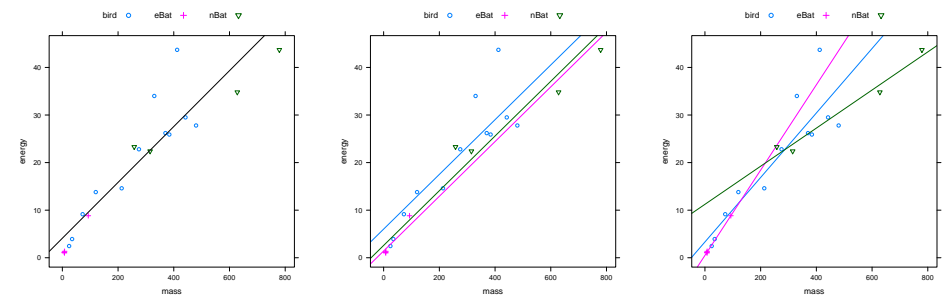
- The scatterplot reveals potential problems with fitting a standard regression model:
 - ▶ Two bird observations appear to be potential outliers;
 - ▶ There is some apparent curvature;
 - ▶ Points with high mass have more variable energy measurements than points with low mass;
- We will, however, fit a few models to illustrate the method and to show how these potential problems can be identified more readily with *residual plots*.

Fitting Models

```
> fit0 = lm(energy ~ mass,
+ data = bats)
> fit1 = lm(energy ~ mass +
+ type, data = bats)
> fit2 = lm(energy ~ mass *
+ type, data = bats)
```

- fit0 is a simple linear regression of energy on mass
- fit1 adds type as an input variable. This has the effect of allowing the intercept to be different for each type.
- fit2 has mass and type and an interaction between them. This allows each type to have its own slope and intercept.

Plots of Fitted Models



Estimated Coefficients

```
> coef(fit0)
```

```
(Intercept)      mass
4.09991727  0.05869642
```

- fit0 shows the intercept and parameter for mass which is the slope.

Estimated Coefficients

```
> coef(fit1)
```

```
(Intercept)      mass  typeeBat  typenBat
6.02197707  0.05749542 -4.60071984 -3.43220829
```

- fit1 shows an intercept for all predictions, a parameter for mass which is the common slope, and then adjustments to be made if the type is eBat or nBat.
- In effect, these are estimated differences of the intercept relative to bird.
- For birds, the intercept is 6.02.
- For echolocating bats the intercept is $6.02 + (-4.6) = 1.42$.
- For non-echolocating bats the intercept is $6.02 + (-3.43) = 2.59$.
- The three lines are parallel and share the common slope 0.0575.

Estimated Coefficients

```
> coef(fit2)
```

```
(Intercept)      mass  typeeBat  typenBat
3.31674159  0.06777464 -2.82275855  7.91064213
mass:typeeBat mass:typenBat
0.02186199  -0.02772895
```

- fit2 shows six estimated coefficients, the intercept and slope for bird and then adjustments to each of these for the other types.
- For birds, the intercept is 3.32 and the slope is 0.0678.
- For echolocating bats the intercept is $3.32 + (-2.82) = 0.494$ and the slope is $0.0678 + (0.0219) = 0.0896$
- For non-echolocating bats the intercept is $3.32 + (7.91) = 11.2$ and the slope is $0.0678 + (-0.0277) = 0.04$

Interpretation of Coefficients

```
> coef(fit2)
```

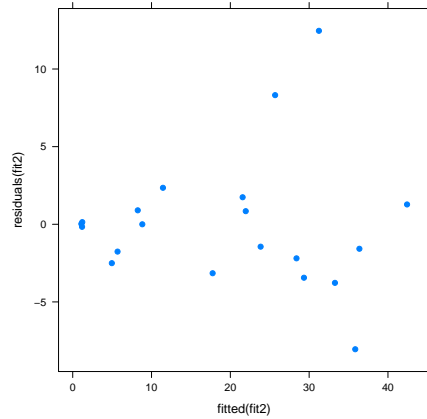
```
(Intercept)      mass  typeeBat  typenBat
3.31674159  0.06777464 -2.82275855  7.91064213
mass:typeeBat mass:typenBat
0.02186199  -0.02772895
```

- The intercept is the predicted energy of a bird at mass 0 — no biological relevance.
- The third coefficient is the estimated difference between the predicted energies for echolocating bats and birds at mass 0.
- Notice that the predicted difference is not the same at all masses.
- This parameter has no biological significance also.
- Similar comments can be made about the non-echolocating bats — in particular, even though the intercept for non-echolocating bats is higher than for birds, at the range of mass where there are both birds and non-echolocating bats, the bird line is *higher*.

Residual Plot

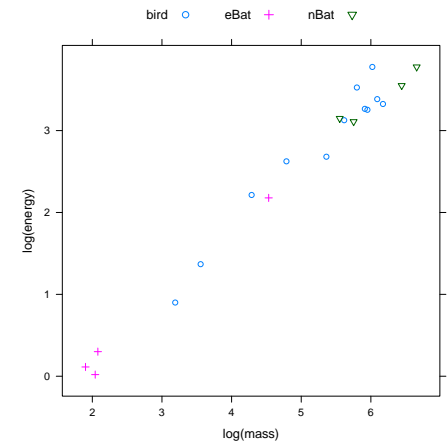
```
> plot(xyplot(residuals(fit2) ~  
+ fitted(fit2), pch = 16))
```

- Residual plot from last fit.
- Notice the fan-shaped pattern.
- Residuals are larger for large mass.
- A transformation may help.



Log Transformed Data

- Log transformation of both variables leads to data that better fits linear model assumptions.



More Analysis

- Do remaining analysis live in R.