Parameter Estimation

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Kiwi Shade Example

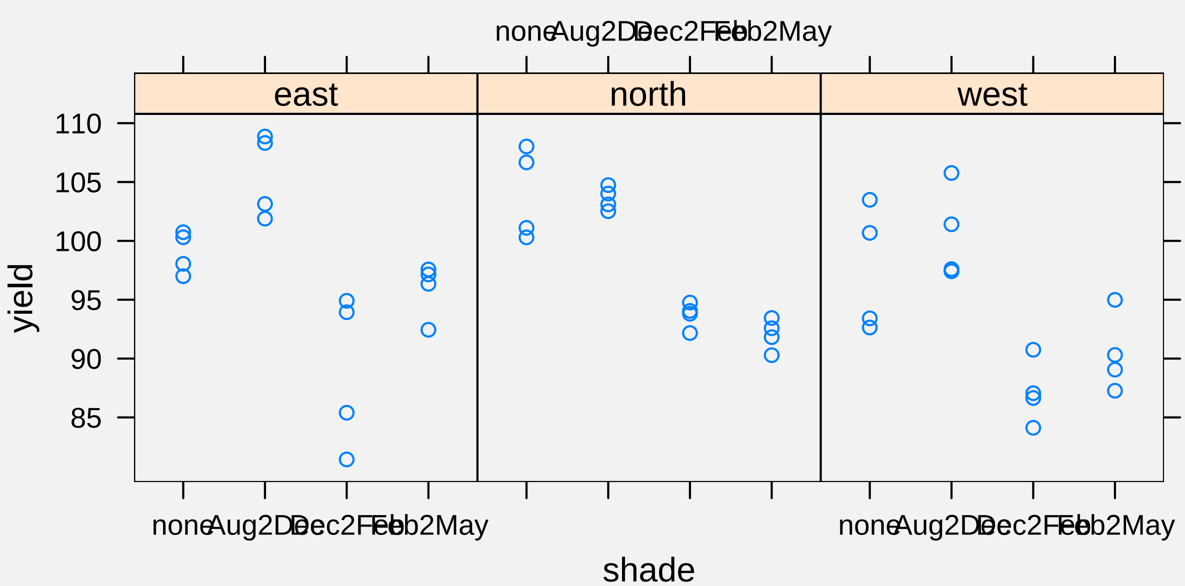
- Continue the kiwi shade example.
- Estimate the shade effects from Model 1.

Data

```
> library(DAAG)
> data(kiwishade)
> attach(kiwishade)
> str(kiwishade)

'data.frame':	48 obs. of  4 variables:
 $ yield: num  101.1 108.0 106.7 100.3  92.6 ...
 $ block: Factor w/ 3 levels "east","north",..: 2 2 2 2 3 3 3 3 1 1 ...
 $ shade: Factor w/ 4 levels "none","Aug2Dec",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ plot : Factor w/ 12 levels "east.Aug2Dec",..: 8 8 8 8 12 12 12 12 4 4 ...
```


Plots



Model 1

```
> library(lme4)
> kiwi1.lmer = lmer(yield ~ shade + (1 | block) + (1 | block:shade))
```

- Treat `block` as a random effect, as in the text.
- Treat `shade` as a fixed effect.
- We are interested in all of the comparisons between shade levels.

Model 1 Summary

```
> summary(kiwi1.lmer)

Linear mixed-effects model fit by REML
Formula: yield ~ shade + (1 | block) + (1 | block:shade)
   AIC   BIC logLik MLdeviance REMLdeviance
 264.0 275.2 -126.0      262.8        252.0
Random effects:
 Groups      Name        Variance Std.Dev.
 block:shade (Intercept)  2.1857  1.4784
 block       (Intercept)  4.0361  2.0090
 Residual                12.1878  3.4911
number of obs: 48, groups: block:shade, 12; block, 3

Fixed effects:
             Estimate Std. Error t value
(Intercept)   100.203      1.758   57.01
shadeAug2Dec    3.031      1.868    1.62
shadeDec2Feb  -10.282      1.868   -5.50
shadeFeb2May   -7.428      1.868   -3.98

Correlation of Fixed Effects:
            (Intr) shdA2D shdD2F
shadeAug2Dc -0.531
shadeDec2Fb -0.531  0.500
shadeFeb2My -0.531  0.500  0.500
```

- For parameter estimation, REML is preferable to ML.
- With the standard parameterization, we have parameters for the mean yield with shade level `none` and differences between `none` and other levels.
- Lets find 95% intervals for all possible pairwise differences.
- This will be somewhat similar to Fisher LSD tests.

Fixed Effects

```
> fe = fixef(kiwi1.lmer)
> fe

 (Intercept) shadeAug2Dec shadeDec2Feb shadeFeb2May
  100.202500     3.030833   -10.281667    -7.428333

> mu = c(fe[1], fe[1] + fe[2:4])
> names(mu)[1] = "none"
> mu

        none shadeAug2Dec shadeDec2Feb shadeFeb2May
   100.20250    103.23333     89.92083     92.77417
```

- R code shows how to find the means for each treatment level.
- Are the pairwise differences significant?

A Parametric Approach

- In this balanced experiment, all of the pairwise differences in shade effects will have the same SE.
- Here it is calculated as 1.868.
- The most appropriate degrees of freedom for t distribution inference is the degrees of freedom associated with plot level.
- We can use the nested factor diagram to see that the best choice for degrees of freedom is 6.
- There are 12 levels in plot and 6 df in variables in which it is nested (1 for the intercept, 3 for shade, 2 for block).

Fixed Model

```
> kiwi1.fixed = lm(yield ~ block * shade)
> anova(kiwi1.fixed)

Analysis of Variance Table

Response: yield
            Df  Sum Sq Mean Sq F value    Pr(>F)
block        2  172.35   86.17  7.0734  0.002565 **
shade        3 1394.51  464.84 38.1553 2.836e-11 ***
block:shade  6  125.57   20.93  1.7178  0.145020
Residuals   36  438.58   12.18
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Fit a fixed effects model just to verify the df.
- See the 6 df for the interaction term.

R Functions for Pairwise Comparisons

```
> source("pairwise.R")
> pairwiseDiff

function (x)
{
    n = length(x)
    nm = names(x)
    np = n * (n - 1)/2
    d = rep(0, np)
    dnames = rep("A", np)
    k = 0
    for (i in 1:(n - 1)) {
        for (j in (i + 1):n) {
            k = k + 1
            d[k] = mu[i] - mu[j]
            dnames[k] = paste(nm[i], "-", nm[j], sep = "")
        }
    }
    names(d) = dnames
    d
}
```

R Functions for Pairwise Comparisons

```
> pairwiseCI

function (d, se, df, conf.level = 0.95)
{
    low = (1 - conf.level)/2
    high = 1 - low
    tmult = qt(high, df)
    a = d - tmult * se
    b = d + tmult * se
    tstat = d/se
    p = 2 * pt(-abs(tstat), df)
    out = data.frame(Diff = d, Low = a, High = b, t = tstat,
        Pvalue = p)
    cat(paste(100 * conf.level, "%", " Confidence Intervals for Pairwise Differences\n",
        sep = ""))
    out
}
```


Confidence Intervals

```
> d = pairwiseDiff(mu)
> ci = round(pairwiseCI(d, 1.868, 6), 4)

95% Confidence Intervals for Pairwise Differences

> ci

                            Diff     Low    High       t Pvalue
none-shadeAug2Dec        -3.0308 -7.6017  1.5400 -1.6225 0.1558
none-shadeDec2Feb        10.2817  5.7108 14.8525  5.5041 0.0015
none-shadeFeb2May         7.4283  2.8575 11.9992  3.9766 0.0073
shadeAug2Dec-shadeDec2Feb 13.3125  8.7417 17.8833  7.1266 0.0004
shadeAug2Dec-shadeFeb2May 10.4592  5.8883 15.0300  5.5991 0.0014
shadeDec2Feb-shadeFeb2May -2.8533 -7.4242  1.7175 -1.5275 0.1775
```

MCMC Approach

```
> set.seed(324)
> kiwi1.mcmc = mcmcsamp(kiwi1.lmer, 10000)
> kiwi1.mcmc[1, ]

   (Intercept)   shadeAug2Dec   shadeDec2Feb   shadeFeb2May   log(sigma^2)
   100.5304568      3.5369449     -9.9892995     -8.5594243      2.3960431
log(blc:.(In)) log(blck.(In))
    -0.4282659      3.5135838

> out = matrix(0, 6, 2)
> out[1, ] = quantile(-kiwi1.mcmc[, 2], c(0.025, 0.975))
> out[2, ] = quantile(-kiwi1.mcmc[, 3], c(0.025, 0.975))
> out[3, ] = quantile(-kiwi1.mcmc[, 4], c(0.025, 0.975))
> out[4, ] = quantile(kiwi1.mcmc[, 2] - kiwi1.mcmc[, 3], c(0.025,
+     0.975))
> out[5, ] = quantile(kiwi1.mcmc[, 2] - kiwi1.mcmc[, 4], c(0.025,
+     0.975))
> out[6, ] = quantile(kiwi1.mcmc[, 3] - kiwi1.mcmc[, 4], c(0.025,
+     0.975))
> round(cbind(out, ci[, 1:3]), 4)

                              1       2    Diff     Low    High
none-shadeAug2Dec       -6.3564  0.2383 -3.0308 -7.6017  1.5400
none-shadeDec2Feb        6.9811 13.5775 10.2817  5.7108 14.8525
none-shadeFeb2May        4.2025 10.6412  7.4283  2.8575 11.9992
shadeAug2Dec-shadeDec2Feb 10.0411 16.5880 13.3125  8.7417 17.8833
shadeAug2Dec-shadeFeb2May  7.2195 13.7467 10.4592  5.8883 15.0300
shadeDec2Feb-shadeFeb2May -6.1151  0.3674 -2.8533 -7.4242  1.7175
```


Histogram (1-4)