

Multi-level Models

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Kiwi Shade Example

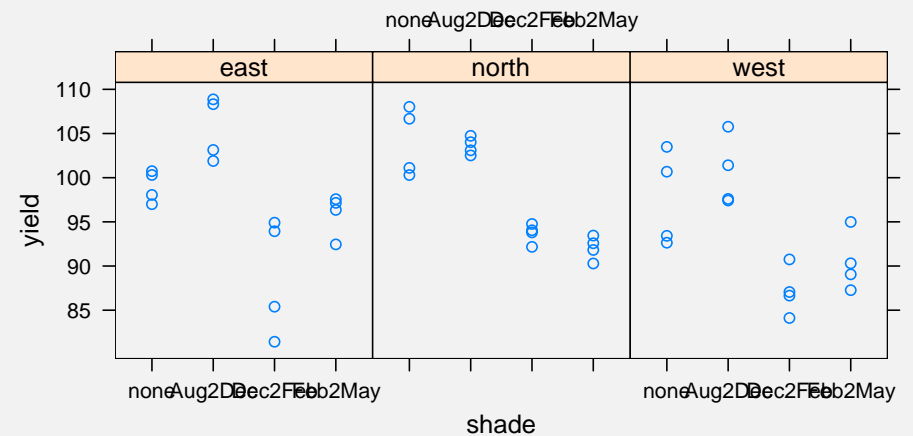
- Designed experiment with four shading treatments.
- Three blocks.
- Four plots within blocks.
- Four vines within plots.
- Shading is applied to all vines within a plot.
- Response is kiwi yield.

Data

```
> library(DAAG)
> data(kiwishade)
> attach(kiwishade)
> str(kiwishade)

'data.frame': 48 obs. of 4 variables:
 $ yield: num 101.1 108.0 106.7 100.3 92.6 ...
 $ block: Factor w/ 3 levels "east","north",...: 2 2 2 2 3 3 3 3 1 1 ...
 $ shade: Factor w/ 4 levels "none","Aug2Dec",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ plot : Factor w/ 12 levels "east.Aug2Dec",...: 8 8 8 8 12 12 12 12 4 4
```

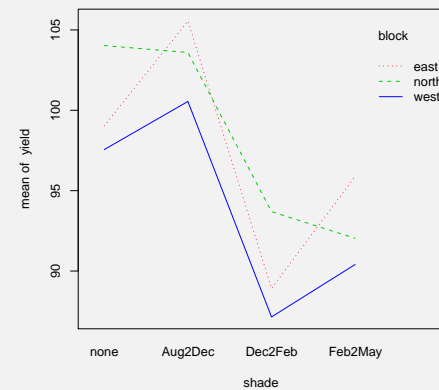
Plots



Observations

- Scatter of points is regular within treatment/block combinations.
- Apparent treatment effect within each plot. (Treatment levels are none, Aug2Dec, Dec2Feb, and Feb2May.)
- Pattern of treatment effect appears to be similar in blocks east and west, but different in north.
- See diagram on page 318 for possible explanation.

Interaction Plot



- Notice that the lines for the East and West blocks are more nearly parallel than North.
- Perhaps the shade effect is more similar within these two blocks than within the North block.
- With only one replicate of shade within each block, we cannot model this apparent interaction as well as a plot level random effect.

Model 1

```
> library(lme4)
> kiwi1.lmer = lmer(yield ~ shade + (1 | block) + (1 | block:shade))
```

- Treat `block` as a random effect, as in the text.
- Treat `shade` as a fixed effect. We care about the specific levels.
- Treat `plot` which is the interaction between `block` and `shade` as a random effect. We must treat this effect as random as it is the level of the experimental unit — shade treatments are applied to plots.
- The individual level variation is at the vine level. We do not need to specify it in the model.

Model 1 Summary

```
> summary(kiwi1.lmer)

Linear mixed-effects model fit by REML
Formula: yield ~ shade + (1 | block) + (1 | block:shade)
      AIC      BIC logLik MLdeviance REMLdeviance
264.0 275.2 -126.0    262.8    252.0
Random effects:
Groups      Name      Variance Std.Dev.
block:shade (Intercept) 2.1857  1.4784
block      (Intercept) 4.0361  2.0090
Residual                    12.1878  3.4911
number of obs: 48, groups: block:shade, 12; block, 3

Fixed effects:
              Estimate Std. Error t value
(Intercept)    100.203     1.758    57.01
shadeAug2Dec     3.031     1.868     1.62
shadeDec2Feb    -10.282     1.868    -5.50
shadeFeb2May    -7.428     1.868    -3.98

Correlation of Fixed Effects:
      (Intr) shdA2D shdD2F
shadeAug2Dc -0.531
shadeDec2Fb -0.531  0.500
shadeFeb2My -0.531  0.500  0.500
```

- Notice that the largest component of variation is at the individual vine level.
- This variation can be explained by biological diversity among the plants. (SD = 3.5.)
- Block level variation and plot level variation are also important (SD = 2.0 for blocks, SD = 1.5 for plots.)
- Some differences between shade levels look like they might be significant.

Model 0

```
> kiwi0.lmer = lmer(yield ~ 1 + (1 | block) + (1 | block:shade))
```

- Set up a null model without shade to test if shade is significant.
- The `block:shade` interaction term is simply a way to specify the plot.

Model 0 Summary

```
> summary(kiwi0.lmer)
```

```
Linear mixed-effects model fit by REML
Formula: yield ~ 1 + (1 | block) + (1 | block:shade)
      AIC      BIC logLik MLdeviance REMLdeviance
288.6 294.3 -141.3   285.6       282.6
Random effects:
Groups      Name      Variance Std.Dev.
block:shade (Intercept) 3.5426e+01 5.9519e+00
block      (Intercept) 6.0910e-09 7.8045e-05
Residual                    1.2182e+01 3.4903e+00
number of obs: 48, groups: block:shade, 12; block, 3

Fixed effects:
              Estimate Std. Error t value
(Intercept)   96.533      1.791    53.91
```

- Now the largest component of variation is at the plot level.
- A true shade effect is possibly being absorbed by the plot level variability.

Likelihood Ratio Test

```
> kiwi0.ml = update(kiwi0.lmer, method = "ML")
> kiwi1.ml = update(kiwi1.lmer, method = "ML")
```

- Likelihood ratio tests using `anova` should be done on models fit using ML instead of REML.
- The `update` command refits the models.
- The actual differences are slight.

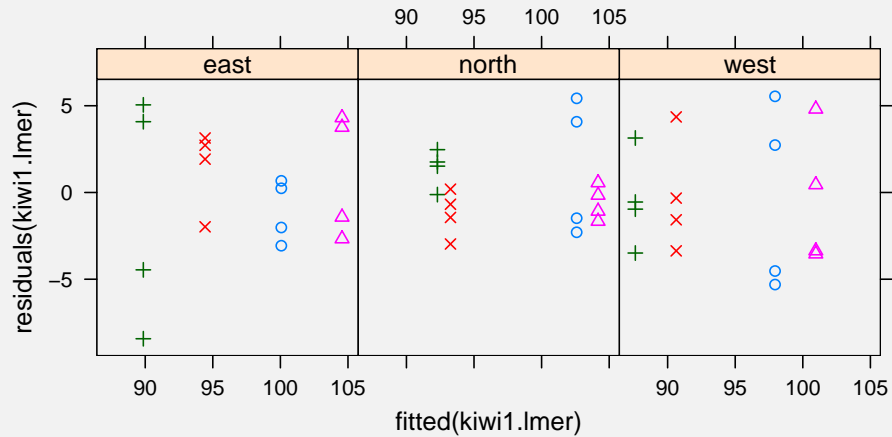
Likelihood Ratio Test

```
> anova(kiwi0.ml, kiwi1.ml)
```

```
Data:
Models:
kiwi0.ml: yield ~ 1 + (1 | block) + (1 | block:shade)
kiwi1.ml: yield ~ shade + (1 | block) + (1 | block:shade)
              Df      AIC      BIC logLik Chisq Chi Df Pr(>Chisq)
kiwi0.ml    3  291.61  297.22 -142.80
kiwi1.ml    6  274.09  285.32 -131.05 23.515    3 3.154e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- The estimated variance components change little, but absolute AIC and BIC values change when using ML and not REML.
- There is very strong evidence that shade has an effect.
- A different analysis is needed to compare specific shade levels.

Residual Plot



Model 2

```
> kiwi2.lmer = lmer(yield ~ shade + block + (1 | block:shade))
```

- To consider differences between blocks, we should model `block` as a fixed effect.
- Without replication of treatments within blocks, we cannot explore the apparent block treatment interaction from the plotted data in which the effect of `shade` looked different in the two south blocks (east and west) than it did in the north block.

Model 2 Summary

```
> summary(kiwi2.lmer)
```

Linear mixed-effects model fit by REML

Formula: `yield ~ shade + block + (1 | block:shade)`

	AIC	BIC	logLik	MLdeviance	REMLdeviance
	255.8	268.9	-120.9	256.3	241.8

Random effects:

Groups	Name	Variance	Std.Dev.
block:shade	(Intercept)	2.1908	1.4801
	Residual	12.1812	3.4902

number of obs: 48, groups: block:shade, 12

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	101.0148	1.6180	62.43
shadeAug2Dec	3.0308	1.8683	1.62
shadeDec2Feb	-10.2817	1.8683	-5.50
shadeFeb2May	-7.4283	1.8683	-3.98
blocknorth	0.9931	1.6180	0.61
blockwest	-3.4300	1.6180	-2.12

Correlation of Fixed Effects:

	(Intr)	shdA2D	shdD2F	shdF2M	blknmr
shadeAug2Dc	-0.577				
shadeDec2Fb	-0.577	0.500			
shadeFeb2My	-0.577	0.500	0.500		
blocknorth	-0.500	0.000	0.000	0.000	
blockwest	-0.500	0.000	0.000	0.000	0.500

- The estimated variance components are very similar to model 1.
- Model 1 and Model 2 cannot be compared with a likelihood ratio test as they are not nested.